

# Tableaux Calculus

## Propositional Logic

A compact version of sequent calculus

# The idea

What's "wrong" with sequent calculus:

Why do we have to copy(?)  $\Gamma$  and  $\Delta$   
with every rule application?

The answer: tableaux calculus.

The idea:

Describe *backward* sequent calculus rule application  
but leave  $\Gamma$  and  $\Delta$  implicit/shared

Comparison:

**Sequent** Proof is a tree labeled by sequents,  
trees grow upwards

**Tableaux** Proof is a tree labeled by formulas,  
trees grow downwards

Terminology: **tableau** = tableaux calculus proof tree

## Tableaux rules (examples)

Notation:  $+F \approx F$  occurs on the right of  $\Rightarrow$   
 $-F \approx F$  occurs on the left of  $\Rightarrow$

<i>S.C.</i>		<i>Tab.</i>		<i>Effect</i>
$\frac{F, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \neg F, \Delta}$	$\rightsquigarrow$	$\frac{+\neg F}{-F}$	$\rightsquigarrow$	$\begin{array}{c} +\neg F \\   \\ -F \end{array}$
$\frac{\Gamma \Rightarrow F, G, \Delta}{\Gamma \Rightarrow F \vee G, \Delta}$	$\rightsquigarrow$	$\frac{+F \vee G}{+F \quad +G}$	$\rightsquigarrow$	$\begin{array}{c} +F \vee G \\   \\ +F \\   \\ +G \end{array}$
$\frac{\Gamma \Rightarrow F, \Delta \quad \Gamma \Rightarrow G, \Delta}{\Gamma \Rightarrow F \wedge G, \Delta}$	$\rightsquigarrow$	$\frac{+F \wedge G}{+F \mid +G}$	$\rightsquigarrow$	$\begin{array}{c} +F \wedge G \\ / \quad \backslash \\ +F \quad +G \end{array}$

Interpretation of tableaux rule

$$\frac{F}{FGH}$$

if  $F$  matches the formula at some node in the tableau  
extend the end of some branch starting at that node  
according to  $FGH$ .

## Example

$$- A \rightarrow B$$

$$- B \rightarrow C$$

$$- A$$

$$+ C$$

$$A \rightarrow B, B \rightarrow C, A \Rightarrow C$$

From tableau to sequents:

- ▶ Every path from the root to a leaf in a tableau represents a sequent
- ▶ The set of all such sequents represents the set of leaves of the corresponding sequent calculus proof

⇒

- ▶ A branch is **closed** (proved) if both  $+F$  and  $-F$  occur on it or  $-\perp$  occurs on it
- ▶ The root sequent is proved if all branches are closed

Algorithm to prove  $F_1, \dots \Rightarrow G_1, \dots$ :

1. Start with the tableau  $-F_1, \dots, +G_1, \dots$ .
2. while there is an open branch do
  - pick some non-atomic formula on that branch,
  - extend the branch according to the matching rule

## Termination

No formula needs to be used twice on the same branch.  
But possibly on *different* branches:

$$\begin{array}{l} +\neg A \wedge \neg B \\ +A \vee B \end{array}$$

A formula occurrence in a tableau can be deleted  
if it has been used in every unclosed branch  
starting from that occurrence

## Tableaux rules

$$\frac{-\neg F}{+F}$$

$$\frac{+\neg F}{-F}$$

$$\frac{-F \wedge G}{-F \\ -G}$$

$$\frac{+F \wedge G}{+F \mid +G}$$

$$\frac{-F \vee G}{-F \mid -G}$$

$$\frac{+F \vee G}{+F \\ +G}$$

$$\frac{-F \rightarrow G}{+F \mid -G}$$

$$\frac{+F \rightarrow G}{-F \\ +G}$$