Chair of Foundations of Software Reliability and Theoretical Computer Science
School of Computation, Information and Technology
Technical University of Munich

Eexam
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**Note:**

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

# Petrinetze

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|---|---|---|---|
| **Exam:** | IN2052 / Endterm | **Date:** | Thursday 7th August, 2025 |
| **Examiner:** | Prof. Dr. Javier Esparza | **Time:** | 14:00 – 15:15 |



## Working instructions

- This exam consists of **8 pages** with a total of **5 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is **45 credits**, including **5 bonus credits**.
  You need **17 credits** to pass.
- Detaching pages from the exam is forbidden.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________ to ________ / Early submission at ________

In all problems, Petri nets do not have arc weights, unless explicitly mentioned.

## Problem 1 Languages of Petri nets (8 credits)

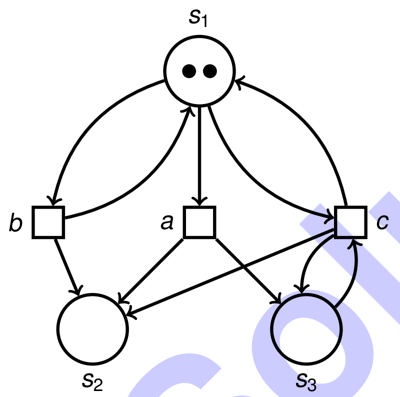
Let $(N, M_0)$ be a Petri net with $N = (S, T, F)$ and let $M$ be a marking. We define

$$L(N, M_0, M) := \{\sigma \in T^* \mid M_0 \xrightarrow{\sigma} M\}$$

to be the set of all firing sequences leading from $M_0$ to $M$.

0 1 2 3 4

a) Give a Petri net $(N, M_0)$ with transition set $T = \{a, b, c\}$ and a marking $M$ such that $L(N, M_0, M) = \{baa, aba, aca\}$ or show that it does not exist.




$M_0 = (2, 0, 0)$ $M = (0, 3, 2)$

0 1 2 3 4

b) Give a Petri net $(N, M_0)$ with transition set $T = \{a, b, c\}$ and a marking $M$ such that $L(N, M_0, M) = \{abcabc\}^* = \{\varepsilon, abcabc, abcabcabcabc, \ldots\}$ or show that it does not exist.

We prove that such a Petri net does not exist.

Assume $L(N, M_0, M) = \{abcabc\}^*$. Since $\varepsilon \in L(N, M_0, M)$, we have $M = M_0$. So $M_0 \xrightarrow{abcabc} M_0$, and from the Marking Equation we get

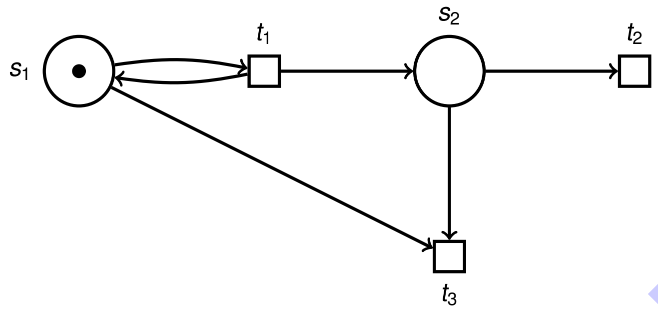
$$\mathbf{N} \cdot \overrightarrow{abcabc} = \mathbf{N} \cdot 2 \cdot \overrightarrow{abc} = 2 \cdot \mathbf{N} \cdot \overrightarrow{abc} = 0$$

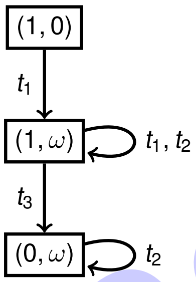
which implies $\mathbf{N} \cdot \overrightarrow{abc} = 0$. It follows $M_0 \xrightarrow{abc} M_0 \xrightarrow{abc} M_0$ and so, in particular, $abc \in L(N, M_0, M)$, contradicting the assumption $L(N, M_0, M) = \{abcabc\}^*$.

## Problem 2 Coverability graph (6 credits)

Give a Petri net $(N, M_0)$ and its coverability graph $G$ such that every node of $G$ has at least one successor, but $(N, M_0)$ has a deadlock.

A Petri net with its coverability graph, where $t_1 t_3$ leads to a deadlock:



# Problem 3 Reductions (9 credits)

a) Given a Petri net $(N, M_0)$, show how to construct a Petri net $(N', M_0')$ **and** a place $s'$ of $N'$ such that:

$$(N, M_0) \text{ is bounded } \textbf{ iff } s' \text{ is bounded in } (N', M_0').$$

Indicate clearly which are the sets of places, transitions and arcs of $N'$, which is the marking $M_0'$, and which is the place $s'$. Sketch a proof of correctness of the construction.

Your construction should be implementable as a program that runs in polynomial time in the size of $(N, M_0)$.

$N'$ is the result of adding to $N$

- a new place $s'$;
- for every place $s$ of $N$, a new transition $t_s$ with preset $\{s\}$ and postset $\{s'\}$. $M_0'$ is the marking that coincides with $M_0$ on all places of $N$ and puts no tokens in $s'$.

We have:

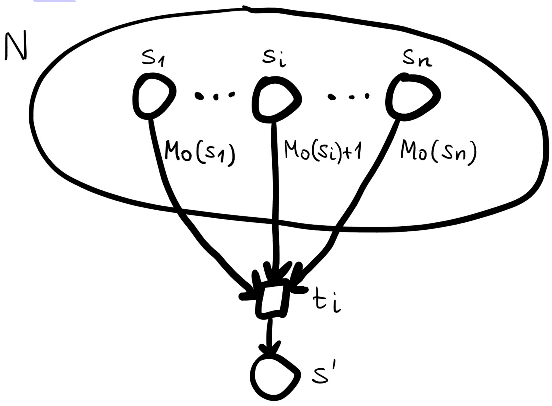
| | |
|---|---|
| | $(N, M_0)$ is unbounded |
| iff | for every $n \geq 0$ some reachable marking of $(N, M_0)$ puts at least $n$ tokens (in total) in the places of $N$ |
| iff | for every $n \geq 0$, some reachable marking of $(N', M_0')$ puts at least $n$ tokens in $s'$ |
| iff | $s'$ is unbounded in $(N', M_0')$. |

b) **Bonus points:** Given a Petri net $(N, M_0)$, show how to construct a **(possibly weighted)** Petri net $(N', M_0')$ **and** a place $s'$ of $N'$ such that:

$$\text{some marking } M \text{ such that } M \neq M_0 \text{ and } M \geq M_0 \text{ is reachable in } (N, M_0) \textbf{ iff}$$
$$\text{some marking } M' \text{ such that } M'(s') > 0 \text{ is reachable in } (N', M_0').$$

Indicate clearly which are the sets of places, transitions and arcs of $N'$, which is the marking $M_0'$, and which is the place $s'$. Sketch a proof of correctness of the construction.

Your construction should be implementable as a program that runs in polynomial time in the size of $(N, M_0)$.

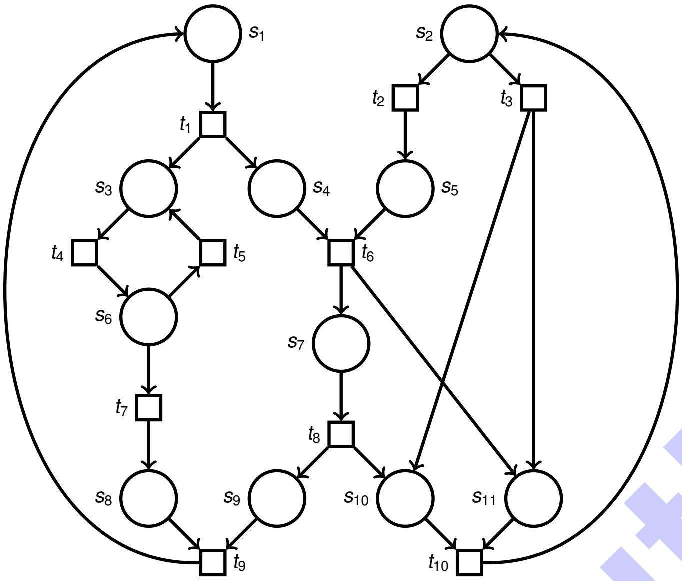



Let $N = (S, T, F)$. $N'$ is the result of adding to $N$ a place $s'$ and one transition $t_s$ for every $s \in S$ such that $W(s, t_s) = M_0(s) + 1$, $W(r, t_s) = M_0(r)$ if $r \in S \setminus \{s\}$, $W(s', t_s) = 0$ and $t_s^\bullet = \{s'\}$. $M_0'$ is the marking that coincides with $M_0$ on all places of $N$ and puts no tokens into $s'$. We have:

| | |
|---|---|
| | there exists a marking $M$ reachable in $(N, M_0)$ such that $M \neq M_0$ and $M \geq M_0$ |
| iff | there exists a marking $M$ reachable in $(N, M_0)$ such that $M \geq M_0 + s$ for some $s \in S$ |
| iff | there exists a marking $M'$ reachable in $(N', M_0')$ such that $M'(s') > 0$. |

# Problem 4 S- and T-invariants (8 credits)

Let $N = (S, T, F)$ be the following net:




a) Give three S-invariants $I_1$, $I_2$, $I_3$ such that $\{I_1, I_2, I_3\}$ is linearly independent.

0 1 2 3 4

| | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ | $s_7$ | $s_8$ | $s_9$ | $s_{10}$ | $s_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I_1$ | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| $I_2$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| $I_3$ | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |

b) Give two T-invariants $J_1$, $J_2$ such that $\{J_1, J_2\}$ is linearly independent.

0 1 2 3 4

| | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ | $t_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $J_1$ | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| $J_2$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

## Problem 5 True or false (14 credits)

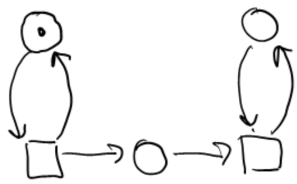
For each of the following statements, decide whether the statement is true. If it is true, give a proof. If it is false, give a counterexample.

**Recall that Petri nets are assumed to be connected and have at least one place and one transition. Therefore, your counterexamples must satisfy these properties.**

0 1 2 3

a) Let $(N, M_0)$ be a T-system. $(N, M_0)$ is deadlock-free **iff** every circuit of $(N, M_0)$ is marked.

This is false. Counterexample:



Not every circuit is marked, but the T-system is deadlock-free.

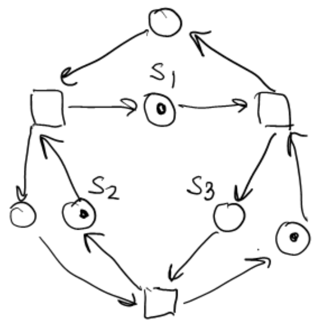
0 1 2 3

b) Let $(N, M_0)$ be a live T-system and let $M$ be a marking of $N$. If $M$ is reachable from $M_0$, then $M_0$ is reachable from $M$.

This is true. By the Reachability Theorem, $M$ is reachable from $M_0$ iff $M \sim M_0$, that is, iff $M$ and $M_0$ agree on all invariants. But the relation "agree on all invariants" is symmetric, that is, $M \sim M_0$ iff $M_0 \sim M$.

c) Let $(N, M_0)$ be a live T-system. $(N, M_0)$ is 1-bounded **iff** $(N, M_0)$ is strongly connected and every simple circuit contains exactly one token.
(Recall that a simple circuit is a circuit that contains each node exactly once.)

This is false. Counterexample:




The simple circuit containing $s_1, s_2, s_3$ has two tokens, but the T-system is live and 1-bounded.

d) Let $(N, M_0)$ be a live free-choice system and let $M$ be a marking such that $M \geq M_0$. Then $(N, M)$ is live.

This is true. By Commoner's theorem, if $(N, M_0)$ is live then every proper siphon of $N$ contains a trap marked at $M_0$. Since $M \geq M_0$, every proper siphon of $N$ contains a trap marked at $M$. By Commoner's theorem, $(N, M)$ is live.

**Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.**