Chair for Foundations of Software Reliability and Theoretical Computer Science
School of Computation, Information and Technology
Technical University of Munich

2316-02-0011-E0001-01

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E0001
Place student sticker here

Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Petrinetze

Exam:	IN2052 / Retake	**Date:**	Monday 30th September, 2024
Examiner:	Prof. Javier Esparza	**Time:**	17:00 – 18:15

	P 1	P 2	P 3	P 4	P 5
I					
II					

Working instructions

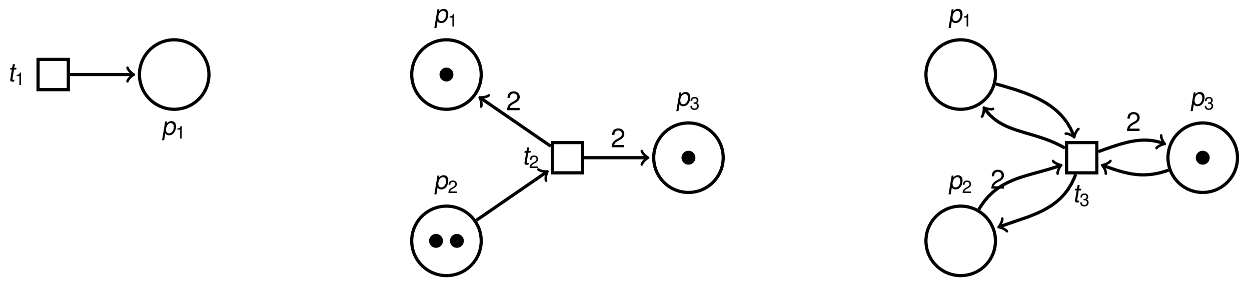
- This exam consists of **8 pages** with a total of **5 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 40 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ____________ to ____________ / Early submission at ____________

☐ Exam empty

Problem 1 **Coverability (8 credits)**

a)* In this exercise you will perform three steps of the backwards coverability algorithm, each on a different net. Remember that such a step $pre_t(M_0)$ consists of computing the minimal marking M_{min} of the upward closed set of markings which can fire t and end up above M_0, i.e. $pre_t(M_0) = \min\{M \mid M \to^t M' \geq M_0\}$.
Consider the following Petri nets with transitions t_1, t_2, t_3.



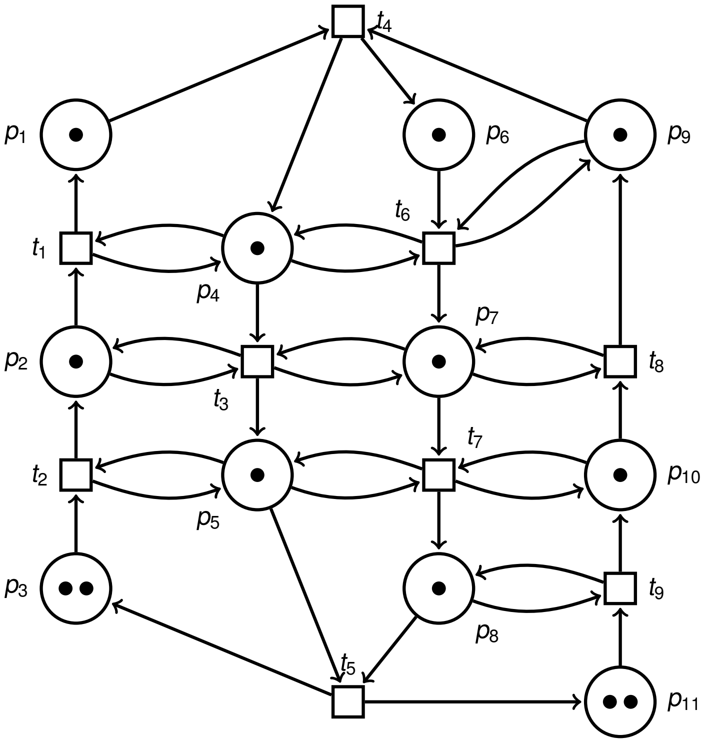
Compute $pre_{t_i}(M_0)$ for each of these nets, where M_0 is the depicted marking.

$pre_{t_1}(0) =$ $\qquad$ $pre_{t_2}(1,2,1) =$ $\qquad$ $pre_{t_3}(0,0,1) =$

b)* Remember that a partial order $\leq$ on a set X is a *well-quasi-order* (wqo) if every infinite sequence $x_1, x_2, \ldots$ in X has an infinite increasing subsequence, i.e. there exist indices $i_1 < i_2 < \ldots$ such that $x_{i_1} \leq x_{i_2} \leq \ldots$
Prove that if $\leq_1$ is a wqo on X_1 and $\leq_2$ is a wqo on X_2, then $\leq_1 \cap \leq_2$ is a wqo on $X_1 \cap X_2$, where $x(\leq_1 \cap \leq_2)y \iff x \leq_1 y$ and $x \leq_2 y$.
You do not have to prove that $\leq_1 \cap \leq_2$ is a partial order again.

Problem 2 Analyzing a Petri net (10 credits)

Consider the following Petri net (N, M_0):



a)* Give a basis of both the vector space of S- and of T-invariants. You are not required (and in fact not intended) to write down the incidence matrix of N.
Hint: It may be helpful to abbreviate e.g. $(x, x, x, x, x, x, y, y, y, y, y)$ with (x^6, y^5).

0 1 2 3 4 5

b)* Is (N, M_0) live? Justify your answer.

☐ True ☐ False

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Problem 3 Construction (4 credits)

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Construct a 1-bounded Petri net satisfying the following specification:

- The set of places is $B_0 \cup B_1$, with $B_0 = \{\mathbf{0}_{n-1}, \ldots \mathbf{0}_0\}$ and $B_1 = \{\mathbf{1}_{n-1}, \ldots \mathbf{1}_0\}$.
- Initially all places of B_0 carry exactly one token, and no place of B_1 carries any tokens.
- Every reachable marking enables at most one transition.
- The net has n transitions and 2^n reachable markings.

Describe with precision the preset and postset of each transition.
A drawing is insufficient here, since the net depends on the parameter n.
Hint: Implement an n-bit counter.

Problem 4 *T*-nets and free-choice nets (10 credits)

We define two notions:

- Given a net N, we denote by $\mathbf{1}_N$ the marking of N that puts one token in each place.
- A Petri net (N, M_0) is *cyclic* if for every marking M reachable from M_0, the marking M_0 is also reachable from M.

Decide if the following statements hold or not. If the statement holds, give a proof. If the statement does not hold, exhibit a counterexample, that is, a net satisfying the premise but not the conclusion.

a)* If N is a T-net, then the T-system $(N, \mathbf{1}_N)$ is cyclic.

☐ True ☐ False

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b)* If $(N, \mathbf{1}_N)$ is a strongly connected free-choice system and some firing sequence $\mathbf{1}_N \xrightarrow{\sigma} M$ of $(N, \mathbf{1}_N)$ fires each transition of N at least once, then $(N, \mathbf{1}_N)$ is live.

☐ True ☐ False

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Problem 5 Reduction (8 credits)

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Consider the following problem, called the *mutual reachability problem*:
Given a *weighted* Petri net N and two markings M_0, M, decide whether *both* $M_0 \rightarrow^* M$ and $M \rightarrow^* M_0$ hold.
Give a polynomial time reduction from coverability for unweighted Petri nets to the mutual reachability problem. Describe the reduction in detail, and sketch correctness.
Observe that your reduction is allowed to introduce weighted arcs.

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Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.

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