Chair for Foundations of Software Reliability and Theoretical Computer Science
School of Computation, Information and Technology
Technical University of Munich

Eexam
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Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
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Petrinetze

Exam:	IN2052 / Retake	**Date:**	Monday 30th September, 2024
Examiner:	Prof. Javier Esparza	**Time:**	17:00 – 18:15

	P 1	P 2	P 3	P 4	P 5
I					
II					

Working instructions

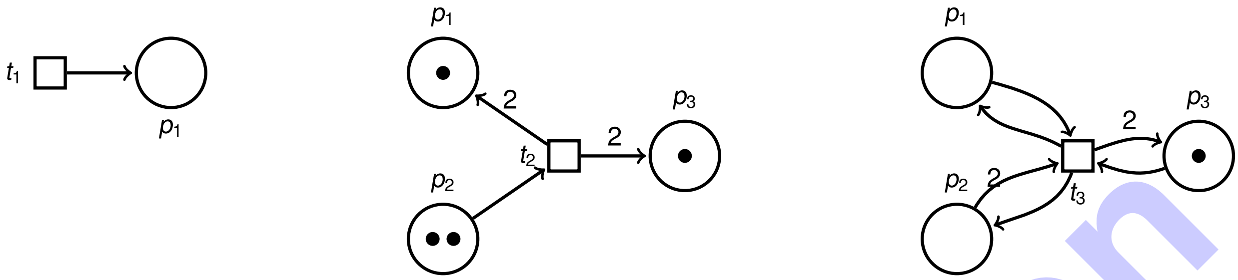
- This exam consists of **8 pages** with a total of **5 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 40 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________	to ________	/	Early submission at ________

Sample Solution

Problem 1 **Coverability (8 credits)**

a)* In this exercise you will perform three steps of the backwards coverability algorithm, each on a different net. Remember that such a step $pre_t(M_0)$ consists of computing the minimal marking M_{min} of the upward closed set of markings which can fire t and end up above M_0, i.e. $pre_t(M_0) = \min\{M \mid M \to^t M' \geq M_0\}$. Consider the following Petri nets with transitions t_1, t_2, t_3.



Compute $pre_{t_i}(M_0)$ for each of these nets, where M_0 is the depicted marking.

$pre_{t_1}(0) = (0)$ $\quad$ $pre_{t_2}(1, 2, 1) = (0, 3, 0)$ $\quad$ $pre_{t_3}(0, 0, 1) = (1, 2, 1)$

b)* Remember that a partial order $\leq$ on a set X is a *well-quasi-order* (wqo) if every infinite sequence $x_1, x_2, ...$ in X has an infinite increasing subsequence, i.e. there exist indices $i_1 < i_2 < ...$ such that $x_{i_1} \leq x_{i_2} \leq ...$

Prove that if $\leq_1$ is a wqo on X_1 and $\leq_2$ is a wqo on X_2, then $\leq_1 \cap \leq_2$ is a wqo on $X_1 \cap X_2$, where $x(\leq_1 \cap \leq_2)y \iff x \leq_1 y$ and $x \leq_2 y$.

You do not have to prove that $\leq_1 \cap \leq_2$ is a partial order again.

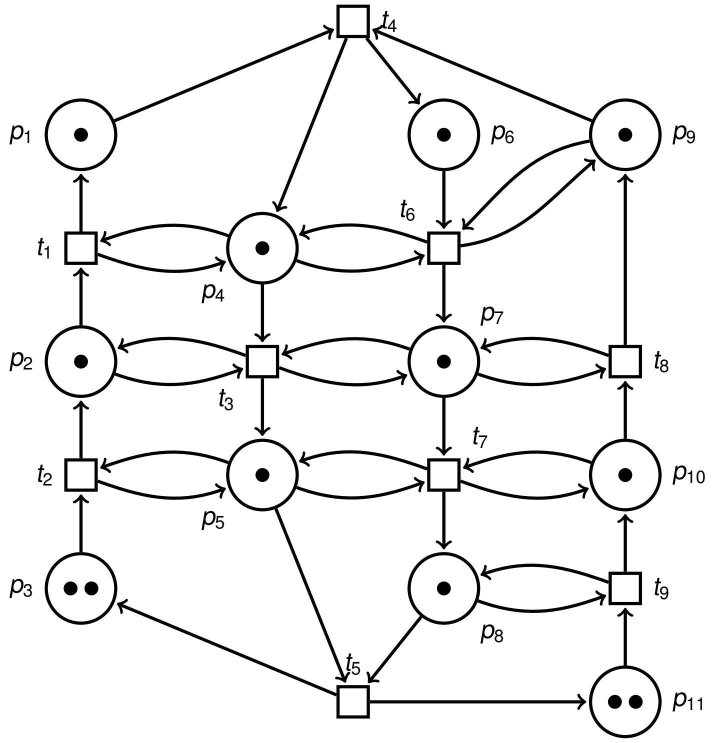
Two possible solutions: 1) Let $x_1, x_2, ...$ be a sequence in $X_1 \cap X_2$. Since $\leq_1$ is a wqo, there exists an infinite subsequence $x_{i_1} \leq_1 x_{i_2} \leq_1 ...$. Since $\leq_2$ is a wqo, there exists a subsubsequence $x_{j_{i_1}} \leq_2 x_{j_{i_2}} \leq_2 ...$. Together we have $x_{j_{i_1}} \leq x_{j_{i_2}} \leq ...$, where $\leq = \leq_1 \cap \leq_2$.

2) By Dickson's Lemma, $(X_1 \times X_2, \leq_1 \times \leq_2)$ is a wqo. Let $x_1, x_2, ...$ be a sequence in $X_1 \cap X_2$. Then $(x_1, x_1), (x_2, x_2), ...$ is a sequence in $X_1 \times X_2$. Since $\leq_1 \times \leq_2$ is a wqo, there exists a subsequence $(x_{i_1}, x_{i_1}), (x_{i_2}, x_{i_2}), ...$ ordered by $\leq_1 \times \leq_2$. Then $x_{i_1}, x_{i_2}, ...$ is an increasing subsequence of $x_1, x_2, ...$ in $X_1 \cap X_2$ as claimed.

Sample Solution

Problem 2 **Analyzing a Petri net (10 credits)**

Consider the following Petri net (N, M_0):



a)* Give a basis of both the vector space of S- and of T-invariants. You are not required (and in fact not intended) to write down the incidence matrix of N.
Hint: It may be helpful to abbreviate e.g. $(x, x, x, x, x, x, y, y, y, y, y)$ with (x^6, y^5).

When only interested in S- and T-invariants, we can remove all edges which exist in both directions, in this case "edges between columns". This leads to a simplified net N, which is in fact a T-system. Therefore a basis for the vector space of T-invariants is $\{(1^9)\}$.
Regarding S-invariants, we observe that places "inside the same column" have to be assigned the same values, and whenever this is the case, all equations for transitions are satisfied, except for t_4 and t_5. So far the shape is hence (x^3, y^2, z^3, w^3). To fulfill the equations for t_4 and t_5 we need $x + w = y + z$. Hence the vector space has dimension 3, and we arrive at the basis e.g. $\{(1^5, 0^6), (0^5, 1^6), (1^3, 0^2, 1^3, 0^3)\}$.

b)* Is (N, M_0) live? Justify your answer.

☐ True ☒ False

It suffices to exhibit a firing sequence reaching a deadlock.
Observe that any marking with tokens only on $p_3, p_{11}, p_4, p_6, p_7$ is a deadlock.
To reach such a marking, fire $t_1 t_8 t_4 t_4 t_5$, emptying all other places.

Sample Solution

Problem 3 Construction (4 credits)

0 1 2 3 4

Construct a 1-bounded Petri net satisfying the following specification:

- The set of places is $B_0 \cup B_1$, with $B_0 = \{\mathbf{0}_{n-1}, \dots \mathbf{0}_0\}$ and $B_1 = \{\mathbf{1}_{n-1}, \dots \mathbf{1}_0\}$.
- Initially all places of B_0 carry exactly one token, and no place of B_1 carries any tokens.
- Every reachable marking enables at most one transition.
- The net has n transitions and 2^n reachable markings.

Describe with precision the preset and postset of each transition.
A drawing is insufficient here, since the net depends on the parameter n.
Hint: Implement an n-bit counter.

Idea: The exact change in the bit representation of a number because of an increment depends on which is the first zero bit. Hence for every bit $i \in \{0, \dots, n-1\}$ we create a transition performing the increment if i is the first zero bit.
Formally, we let $T := \{t_i \mid 0 \leq i \leq n-1\}$ and set

$$F := \bigcup_{i=0}^{n-1} \left(\{(\mathbf{1}_j, t_i), (t_i, \mathbf{0}_j) \mid 0 \leq j < i\} \cup \{(\mathbf{0}_i, t_i), (t_i, \mathbf{1}_i\} \right).$$

An alternative notation is to define for all i the preset and postset of t_i via
${}^\bullet t_i := \{\mathbf{1}_j \mid 0 \leq j < i\} \cup \{\mathbf{0}_i\}$ and $t_i^\bullet := \{\mathbf{0}_j \mid 0 \leq j < i\} \cup \{\mathbf{1}_i\}$.

Sample Solution

Problem 4 T-nets and free-choice nets (10 credits)

We define two notions:

- Given a net N, we denote by $\mathbf{1}_N$ the marking of N that puts one token in each place.
- A Petri net (N, M_0) is *cyclic* if for every marking M reachable from M_0, the marking M_0 is also reachable from M.

Decide if the following statements hold or not. If the statement holds, give a proof. If the statement does not hold, exhibit a counterexample, that is, a net satisfying the premise but not the conclusion.

a)* If N is a T-net, then the T-system $(N, \mathbf{1}_N)$ is cyclic.

☒ True ☐ False

0 1 2 3 4 5

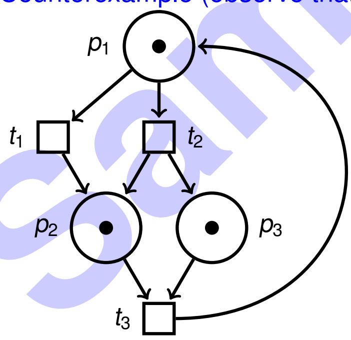
Since $\mathbf{1}_N$ marks every place, it also marks every circuit of N. So, by the Liveness Theorem for T-systems, $(N, \mathbf{1}_N)$ is live. By the Reachability Theorem for T-systems, for any two markings M_1, M_2 of N, M_2 is reachable from M_1 iff M_1 and M_2 agree on all S-invariants of N. Therefore, M_1 is reachable from M_2 iff M_2 is reachable from M_1. So $(N, \mathbf{1}_N)$ is cyclic.

b)* If $(N, \mathbf{1}_N)$ is a strongly connected free-choice system and some firing sequence $\mathbf{1}_N \xrightarrow{\sigma} M$ of $(N, \mathbf{1}_N)$ fires each transition of N at least once, then $(N, \mathbf{1}_N)$ is live.

☐ True ☒ False

0 1 2 3 4 5

Counterexample (observe that due to theorems in the lecture, it cannot be an S- or T-system):



Removing t_1, this would be a nice T-system. But due to t_1, we can fire $t_3 t_1 t_1$ and reach a deadlock.

Sample Solution

Problem 5 Reduction (8 credits)

0 1 2 3 4 5 6 7 8

Consider the following problem, called the *mutual reachability problem*:
Given a *weighted* Petri net N and two markings M_0, M, decide whether *both* $M_0 \to^* M$ and $M \to^* M_0$ hold.
Give a polynomial time reduction from coverability for unweighted Petri nets to the mutual reachability problem. Describe the reduction in detail, and sketch correctness.
Observe that your reduction is allowed to introduce weighted arcs.

Let N, M_0, M as in the coverability problem be given. Idea: We make three main changes: 1) Upon being $\geq M$, we move to a new "phase". 2) In the new phase, we are allowed to remove any tokens from other places p. 3) We are allowed to end the new phase, readding M_0 to the net N.
Formally, let $N = (P, T, F)$. We define $P' := P \cup \{p_{new}\}$, $T' := T \cup \{t_p \mid p \in P\} \cup \{t_1, t_2\}$. We set the arcs as follows: ${}^\bullet t_1 := M$, $t_1^\bullet := \{p_{new}\}$. ${}^\bullet t_2 := \{p_{new}\}$, $t_2^\bullet := M_0$. For all $p \in P$ we set ${}^\bullet t_p := \{p, p_{new}\}$, $t_p^\bullet := \{p_{new}\}$.
For all transitions $t \in T$ we leave the arcs as they were.
Finally, we set $M_0' := M_0$ and $M' := \{p_{new}\}$.
Correctness: "$\Rightarrow$": Since M_0 can cover M via some σ, we can at $M_0' = M_0$ use the firing sequence $\sigma t_1 \mathit{pay}$ to reach M', where pay is a firing sequence removing excess tokens via the t_p transitions.
From M' we can use t_2 to reach M_0'.
"$\Leftarrow$": Let σ be a firing sequence reaching M' from M_0'. Then t_1 must have occurred to produce p_{new}.
Before the first occurrence of t_1, no other new transition could be fired. Hence the prefix of σ before the first occurrence of t_1 covered the marking ${}^\bullet t_1 = M$ using only transitions of the original net N.

Sample Solution

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.

Sample Solution

Sample Solution