Chair for Foundations of Software Reliability and Theoretical Computer Science
School of Computation, Information and Technology
Technical University of Munich

E0001
Place student sticker here

Note:

- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Petrinetze

Exam: IN2052 / Endterm **Date:** Thursday 1st August, 2024
Examiner: Prof. Javier Esparza **Time:** 11:30 – 12:45

	P 1	P 2	P 3
I			
II			

Left room from __________ to __________

from __________ to __________

Early submission at __________

Notes ____________________

Endterm

Petrinetze

Prof. Javier Esparza
Chair for Foundations of Software Reliability and Theoretical Computer Science
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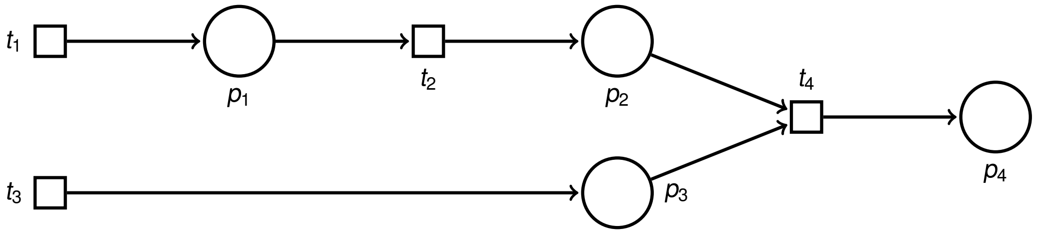
Working instructions

- This exam consists of **4 pages** with a total of **3 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 43 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English $\leftrightarrow$ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Problem 1 Coverability Graphs (10 credits)

0 1 2 3 4

a)* Construct the coverability graph of the following Petri net $(\mathcal{N}, M_0)$, where $M_0 = \mathbf{0}$, that is, the initial marking puts zero tokens in all places.

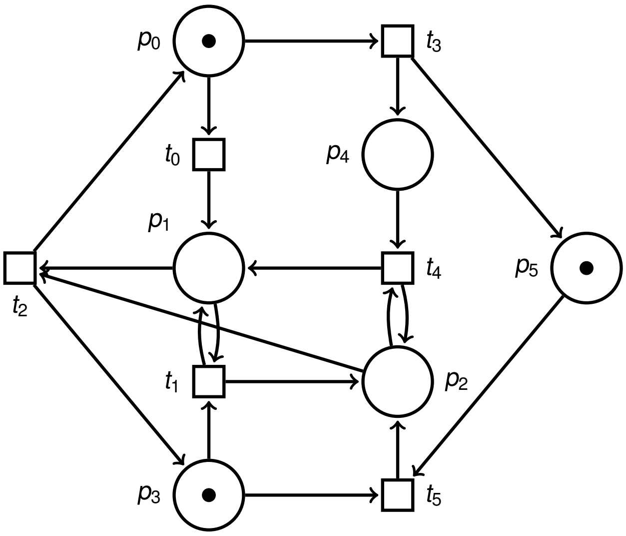


0 1 2 3 4 5 6

b)* Prove that the coverability graph of a Petri net $(\mathcal{N}, M_0)$ with $M_0 = \mathbf{0}$ contains at most 2^n nodes, where n is the number of places of $\mathcal{N}$.

Problem 2 **Analyzing a Petri Net (15 credits)**

Consider the following net $\mathcal{N}$:



Answer the following questions about the Petri net $\mathcal{N}$.
If the object exists, it is enough to give one, otherwise prove that such an object cannot exist.

a)* Is there an S-invariant I with $I(p_0) > 0$?

☐ True ☐ False

0 1 2 3

b)* Is there an S-invariant I with $I(p_5) > 0$?

☐ True ☐ False

0 1 2 3

c)* Is there a positive T-invariant?

☐ True ☐ False

0 1 2 3

d)* Give a smallest siphon containing p_5. (That is, a siphon containing p_5 that does not properly contain any other siphon P'' with $p_5 \in P''$.)

0 1 2 3

e)* Fact: Every minimal proper siphon of $(\mathcal{N}, M_0)$ is an initially marked trap. (You do not have to prove this fact.) Using this fact, prove that $(\mathcal{N}, M_0)$ is deadlock-free.

0 1 2 3

Problem 3 Reductions (18 credits)

a)* Let $\mathcal{N} = (S, T, F)$ be a net with $T = \{t_1, \ldots, t_m\}$. A sequence $\sigma \in T^*$ is *ordered* if σ belongs to the language of the regular expression $t_1^* t_2^* \cdots t_m^*$. For example, $t_2 t_3$ and $t_1 t_1 t_2 t_4 t_4 t_4$ are ordered, but $t_2 t_1$ is not ordered.
The *ordered* reachability problem is defined as follows: Given a net $\mathcal{N} = (S, T, F)$ with a set $T = \{t_1, \ldots, t_m\}$ of transitions, and two markings M_0, M of $\mathcal{N}$, is there an ordered firing sequence σ such that $M_0 \xrightarrow{\sigma} M$?
Prove that the ordered reachability problem can be reduced in polynomial time to the reachability problem. Describe the reduction in detail, and *sketch* the proof that the reduction is correct.

b)* Let $\mathcal{N}$ be a net without weights and let M_0, M be two markings of $\mathcal{N}$ satisfying $M_0, M \geq \mathbf{1}$, that is, satisfying that $M_0(s) \geq 1$ and $M(s) \geq 1$ hold for every place s of $\mathcal{N}$.
Prove: If the marking equation $M = M_0 + N \cdot X$ has a nonnegative integer (i.e. $\mathbb{N}$) solution X, then there exists a natural number $\lambda \in \mathbb{N}_{>0}$ such that λM is reachable from λM_0.
Hint: Let σ have parikh vector X, and prove $i \cdot M_0 + (\lambda - i)M \xrightarrow{\sigma} (i - 1) \cdot M_0 + (\lambda - i + 1)M$ for all $1 \leq i \leq \lambda$.

c)* Prove that the assumption $M \geq \mathbf{1}$ is necessary, by giving a net $\mathcal{N}$ without weights and two markings M_0, M such that $M_0 \geq \mathbf{1}$ and the marking equation $M = M_0 + N \cdot X$ has a nonnegative integer solution, but there does not exist a natural number $\lambda \in \mathbb{N}_{>0}$ such that λM is reachable from λM_0.