Tableaux Calculus
Propositional Logic

A compact version of sequent calculus

The idea

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What's "wrong" with sequent calculus:

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Describe *backward* sequent calculus rule application
but leave Γ and Δ implicit/shared

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Sequent	Proof is a tree labeled by sequents, trees grow upwards
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Terminology: tableau = tableaux calculus proof tree

Tableaux rules (examples)

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Notation: $+F$

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S.C.	*Tab.*	*Effect*

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Interpretation of tableaux rule

$$\frac{F}{FGH}$$

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according to FGH.

Example

$\Rightarrow$

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$A \to B, \qquad \Rightarrow$

Example

$$- A \to B$$

$A \to B,$ $\Rightarrow$

Example

$$- A \to B$$

$$A \to B, B \to C, \quad \Rightarrow$$

Example

$$- A \to B$$
$$- B \to C$$

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Example

$$- A \to B$$
$$- B \to C$$

$$A \to B, B \to C, A \Rightarrow$$

Example

$$
\begin{gathered}
- A \to B \\
- B \to C \\
- A
\end{gathered}
$$

$A \to B, B \to C, A \Rightarrow$

Example

$$- A \to B$$
$$- B \to C$$
$$- A$$

$A \to B, B \to C, A \Rightarrow C$

Example

$$
\begin{array}{c}
- A \to B \\
- B \to C \\
- A \\
+ C
\end{array}
$$

$A \to B, B \to C, A \Rightarrow C$

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Algorithm to prove $F_1, \ldots \Rightarrow G_1, \ldots$:

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1. Start with the tableau $-F_1, \ldots, +G_1, \ldots$.

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1. Start with the tableau $-F_1, \ldots, +G_1, \ldots$.
2. while there is an open branch do

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1. Start with the tableau $-F_1, \ldots, +G_1, \ldots$.
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1. Start with the tableau $-F_1, \ldots, +G_1, \ldots$.
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   pick some non-atomic formula on that branch,
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No formula needs to be used twice on the same branch.

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But possibly on *different* branches:

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$$+A \vee B$$

A formula occurrence in a tableau can be deleted
if it has been used in every unclosed branch
starting from that occurrence

Tableaux rules

$$\frac{-\neg F}{+F} \qquad \frac{+\neg F}{-F}$$

$$\frac{-F \wedge G}{\begin{array}{c}-F\\-G\end{array}} \qquad \frac{+F \wedge G}{+F \mid +G}$$

$$\frac{-F \vee G}{-F \mid -G} \qquad \frac{+F \vee G}{\begin{array}{c}+F\\+G\end{array}}$$

$$\frac{-F \to G}{+F \mid -G} \qquad \frac{+F \to G}{\begin{array}{c}-F\\+G\end{array}}$$