

EXERCISE SHEET: QUANTIFIER ELIMINATION

Exercise 1: Fourier-Motzkin Elimination

Apply the Fourier–Motzkin Elimination to check the following sentences:

1. $\exists x \exists y (2 \cdot x + 3 \cdot y = 7 \wedge x < y \wedge 0 < x)$
2. $\exists x \exists y (3 \cdot x + 3 \cdot y < 8 \wedge 8 < 3 \cdot x + 2 \cdot y)$

Use $\iff$ if two formulas are logically equivalent and $\iff_{R_+}$ if the equivalence requires the theory R_+ .

Exercise 2: Ferrante-Rackoff Elimination

Apply the Ferrante–Rackoff Elimination to check the validity of the following sentence:

$$\exists x (\exists y (x = 2 \cdot y) \rightarrow (2 \cdot x \geq 0 \vee 3 \cdot x < 2))$$

Exercise 3: Presburger Arithmetic

Using quantifier elimination check whether the following sentence belongs to Presburger arithmetic.

$$\forall x \exists y ((x < 2y + 1 \wedge 2y < x + 1) \vee (x < 2y + 2 \wedge 2y < x))$$

Exercise 4: Completeness

Which of the following theories are complete? Justify your answers.

1. Presburger arithmetic,
2. Theory of linear orders,
3. Theory of dense linear orders,
4. Group theory.