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Exercise sheet: Propositional Proof Theory

Exercise 1: Natural Deduction

Prove the following formula using natural deduction.

$$\neg(\forall x(\exists y(\neg P(x) \land P(y))))$$

Exercise 2: Sequent Calculus

Prove the following formulae in sequent calculus:

1. $\neg\exists x P(x) \to \forall x \neg P(x)$
2. $(\forall x(P \lor Q(x))) \to (P \lor \forall x Q(x))$

Exercise 3: Natural Deduction can Simulate Sequent Calculus II

In exercise 6.2 we proved that if $\Gamma \vdash_G \Delta$ then $\Gamma \vdash_N \bigvee \Delta$ for formulae in propositional logic. Augment your proof by the new cases for FOL.

Exercise 4: Counterexamples from Sequent Calculus

Consider the statement $\forall x(P(x) \to \neg P(f(x)))$.

1. What happens when trying to prove the validity of this formula in sequent calculus?
2. How can we derive a countermodel from the proof tree?
3. Is there a smaller countermodel?