

## EXERCISE SHEET: RESOLUTION AND UNIFICATION

### Exercise 1: Exclusive Barbers' Club

There is a town with an exclusive Barbers' club. A barber is a member of the club if and only if he doesn't shave people who shave him (the barber). The barber Arturo claims, that he shaves all members of the club and no one else.

- (a) Give an informal proof that Arturo is lying.
- (b) Formalize the membership property and Arturo's claim.
- (c) Use ground resolution to show that the conjunction of both formulae is unsatisfiable.

### Exercise 2: Herbrand's theorem

The Herbrand's theorem as we have seen it in Lecture 10, holds only for formulas of first-order theory without equality. Give a formula in first-order logic with equality for which Herbrand's theorem does not hold.

### Exercise 3: Occurrence check

During the unification algorithm it is checked whether a term contains the variable it is replacing. This is called the "occurrence check". Assume now a unification algorithm which omits this occurrence check. Then, give a set  $\mathbb{L} = \{L_1, L_2\}$  such that  $L_1$  and  $L_2$  do not share a variable and cannot be unified but the (modified) unification algorithm gives either that  $\mathbb{L}$  can be modified or runs into an infinite loop.

### Exercise 4: Happy Dragons

**Note:** This exercise requires the resolution algorithm for predicate logic, which you will only cover in the lecture on Tuesday, July 1st.

Express the following facts by formulas in predicate logic.

- (a) Every dragon is happy if all its children can fly.
- (b) Green dragons can fly.
- (c) A dragon is green if it is a child of at least one green dragon.

Prove by resolution that the conjunction of these three statements implies the following: all green dragons are happy.