EXERCISE SHEET: RESOLUTION AND DPLL

Exercise 1: DPLL

- 1. Apply the DPLL algorithm to the following sets of clauses. Use OLR first, then PLR. Whenever it is possible to choose multiple literals, choose the variable with the smallest index first, and choose x_i before $\neg x_i$.
 - a) $\{\{\neg x_1, x_4, x_5\}, \{x_2, x_3\}, \{\neg x_2, x_3\}, \{x_1, x_4, x_5\}, \{\neg x_3, x_4, \neg x_5\}, \{\neg x_4, x_5\}, \{\neg x_4, \neg x_5\}\}$
 - b) $\{\{\neg x_1, x_2, x_3\}, \{\neg x_2, x_3\}, \{\neg x_3, x_4, x_5\}, \{\neg x_4, \neg x_5\}, \{\neg x_4, x_5\}, \{x_4, \neg x_5\}, \{x_1, \neg x_2\}\}$
- 2. Apply the CDCL algorithm using resolution for clause learning to the same formulae.

Exercise 2: Input resolution

Let $\mathbb{C} = \{C_1, \ldots, C_n\}$ be a set of clauses. We say a sequence $\langle B_0, B_1, \ldots, B_k \rangle$ is an *input* resolution of B_k from \mathbb{C} if the following holds:

- (i) $B_0 \in \mathbb{C}$, and
- (ii) $B_{\ell+1}$ is the resolvent of B_{ℓ} and one clause from \mathbb{C} .
- 1. Show that there is a resolution of \Box but no input resolution of \Box from

$$F = \{\{\neg z, x\}, \{\neg x, \neg y\}, \{y, z\}, \{z, \neg y, x\}, \{y, \neg x\}\}.$$

2. Show that there is an input resolution of \Box from

$$\{\{\neg t, \neg y\}, \{\neg y, z\}, \{\neg x, \neg z, t\}, \{\neg x, y\}, \{x\}\}.$$

3. Prove that input resolution is complete for Horn formulae by showing that there is an input resolution of □ from any unsatisfiable set of clauses where every clause contains at most one positive literal.¹

Hint

Consider a Horn formula as a conjunction of rules $\bigwedge A \to H$ with A being a set of propositional variables and H being either a propositional variable or 0, i.e. the procedural reading of Horn formulae. Try to consider an "inverse" marking algorithm which starts from 0 and marks propositional variables on the left hand side of implications until a proof why 0 is implied by any possible assignment which sets the right hand sides of rules $1 \to H \equiv \bigwedge \emptyset \to H$ to true is found. Then, frame the executed steps in terms of input resolution.

¹Note that input resolution is not complete in general because F from 1. is unsatisfiable but there is no input resolution proof of that fact.