Verification

Verification

- We use languages to describe the implementation and the specification of a system.
- We reduce the verification problem to language inclusion between implementation and specification

```
1  while x = 1 do
2      if y = 1 then
3          x ← 0
4      y ← 1 − x
5  end
```

- Configuration: triple $[l, n_x, n_y]$ where
  - l is the current value of the program counter, and
  - n_x, n_y are the current values of x, y

Examples: [1,1,1], [5,0,1]

- Initial configuration: configuration with $l = 1$
- Potential execution: finite or infinite sequence of configurations

Examples: [1,1,1][4,1,0]

[2,1,0][5,1,0]

[1,1,0][2,1,0][4,1,0][1,1,0]

```
1  while x = 1 do
2      if y = 1 then
3          x ← 0
4      y ← 1 − x
5  end
```

- Execution: potential execution starting at an initial configuration, and where configurations are followed by their „legal successors“ according to the program semantics.

  Examples: [1,1,1][2,1,1][3,1,1][4,0,1][1,0,1][5,0,1]

  [1,1,0][2,1,0][4,1,0][1,1,0]

- Full execution: execution that cannot be extended (either infinite or ending at a configuration without successors)

Verification as a language problem

- Implementation: set E of executions
- Specification:
  - subset P of the potential executions that satisfy a property , or
  - subset V of the potential executions that violate a property
- Implementation satisfies specification if :
  - $E \subseteq P$, or
  - $E \cap V = \emptyset$.
- If E and P regular: inclusion checkable with automata
- If E and V regular: disjointness checkable with automata

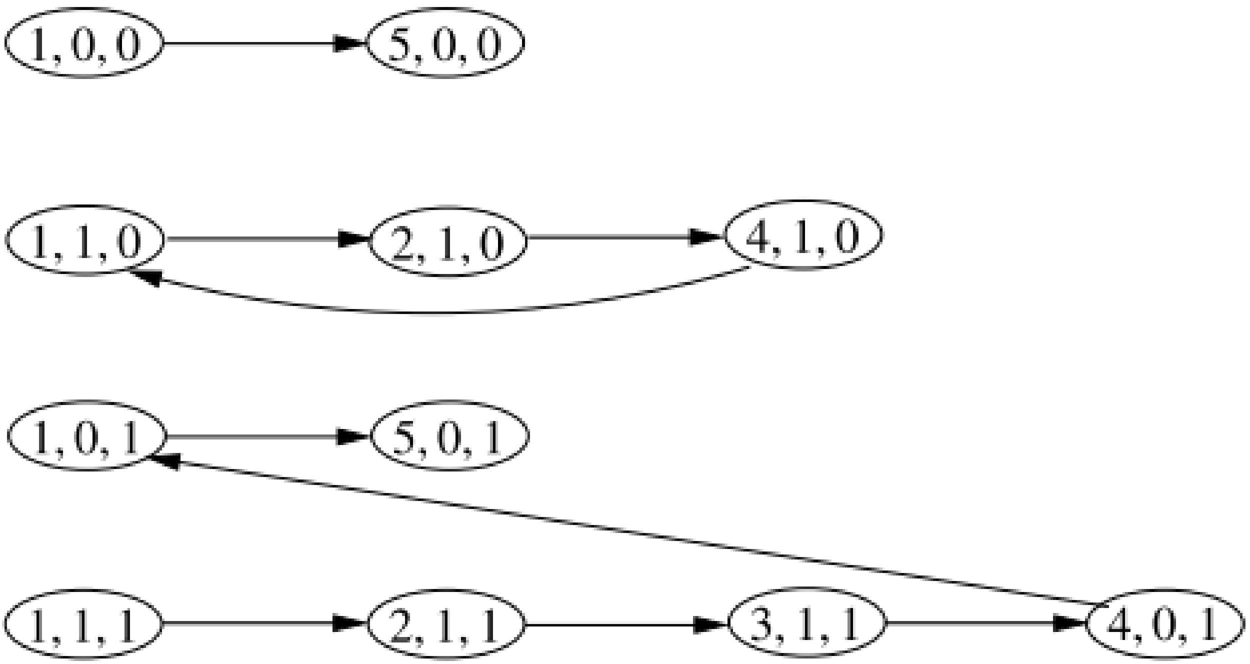
Verification as a language problem

- Implementation: set E of executions
- Specification:
  - subset P of the potential executions that satisfy a property , or
  - subset V of the potential executions that violate a property
- Implementation satisfies specification if :
  - $E \subseteq P$, or
  - $E \cap V = \emptyset$.
- If E and P regular: inclusion checkable with automata
- If E and V regular: disjointness checkable with automata

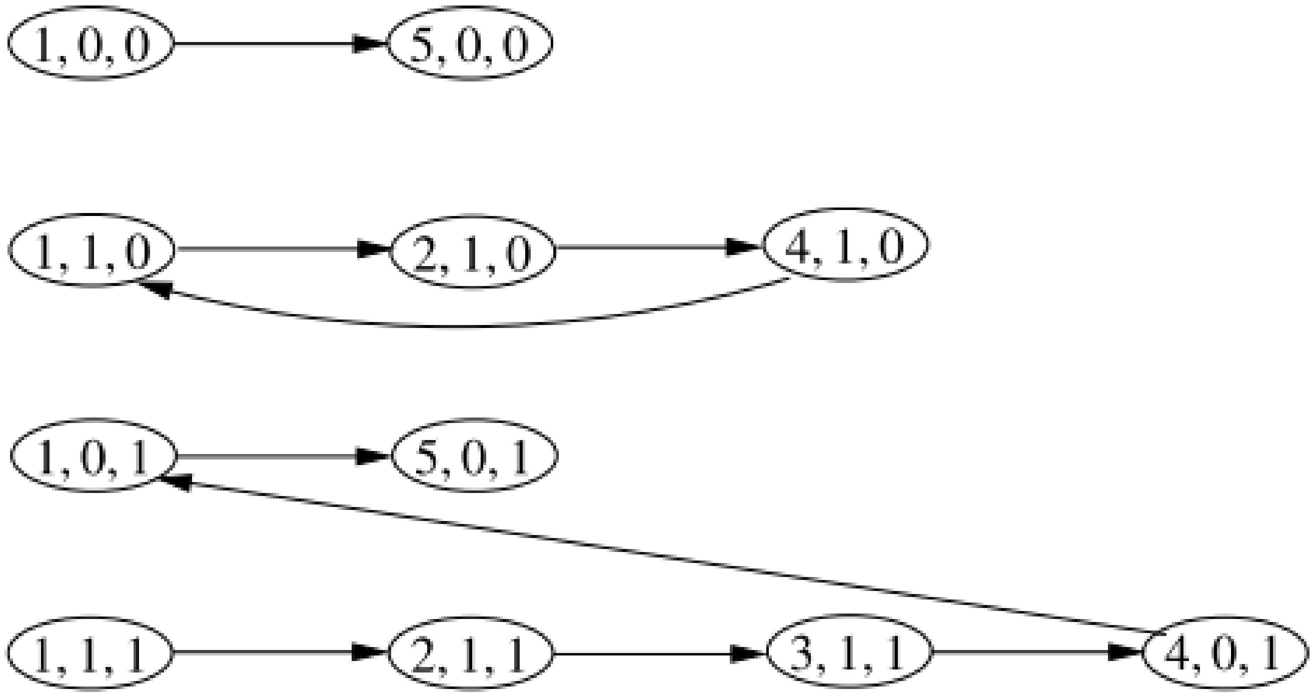
- How often is the case that E, P, V are regular?

System NFA

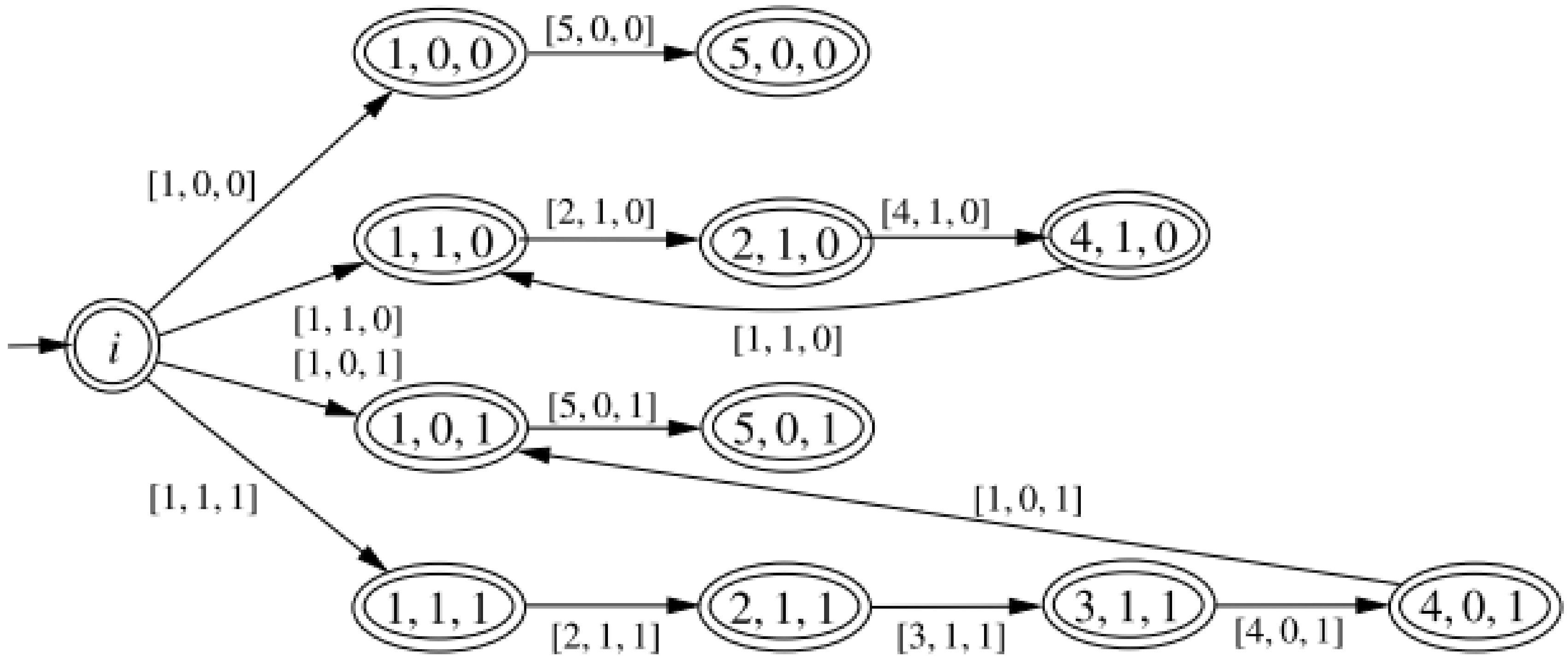
```
while x = 1 do
    if y = 1 then
        x ← 0
    y ← 1 − x
end
```



System NFA



System NFA



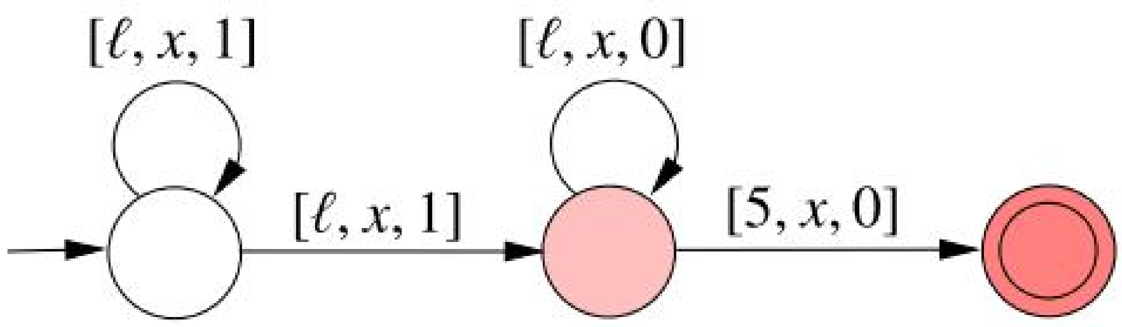
1, 0, 0
5, 0, 0
[5, 0, 0]
[1, 0, 0]
1, 1, 0
2, 1, 0
4, 1, 0
[2, 1, 0]
[4, 1, 0]
i
[1, 1, 0]
[1, 1, 0]
[1, 0, 1]
1, 0, 1
5, 0, 1
[5, 0, 1]
[1, 0, 1]
[1, 1, 1]
1, 1, 1
2, 1, 1
3, 1, 1
4, 0, 1
[2, 1, 1]
[3, 1, 1]
[4, 0, 1]

Property NFA

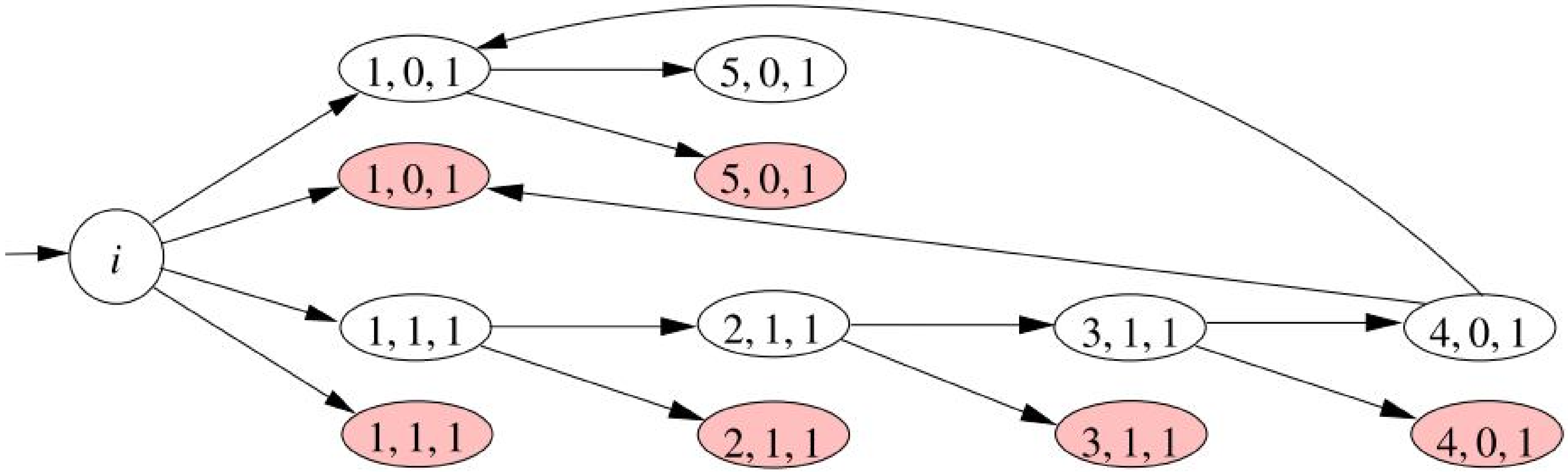
- Is there a full execution such that
  - initially $y = 1$,
  - finally $y = 0$, and
  - y never increases?
- Set of potential executions for this property:

$$[l, x, 1][l, x, 1]^* \ [l, x, 0]^* \ [5, x, 0]$$

- Automaton for this set:



Intersection of the system and property NFAs



- Automaton is empty, and so no execution satisfies the property

Another property

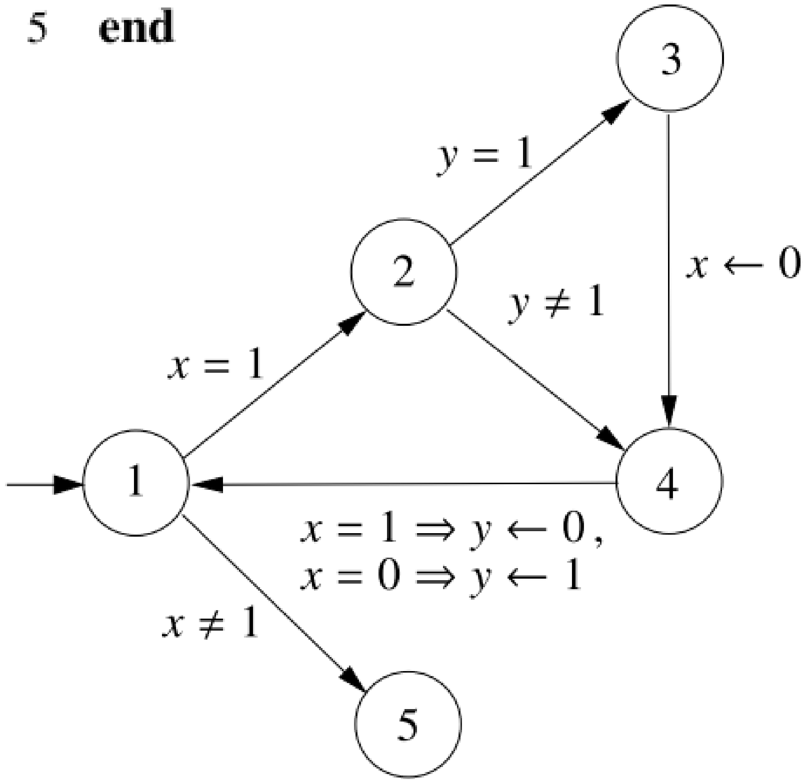
- Is the assignment $y \leftarrow x - 1$ redundant?
- Potential executions where the assignment is executed at least once **and** it changes the value of y:

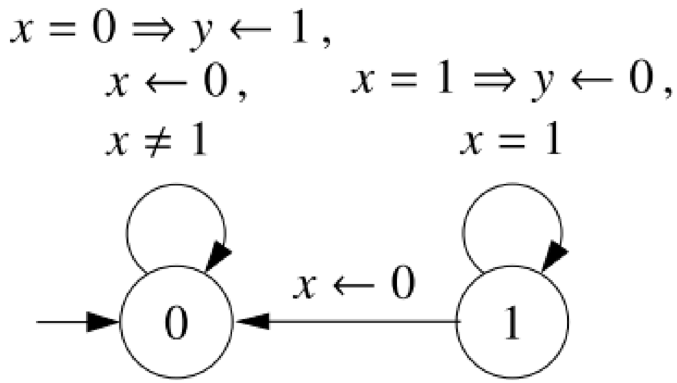
$$[l, x, y]^*([4, x, 0][1, x, 1] + [4, x, 1][1, x, 0])\ [l, x, y]^*$$

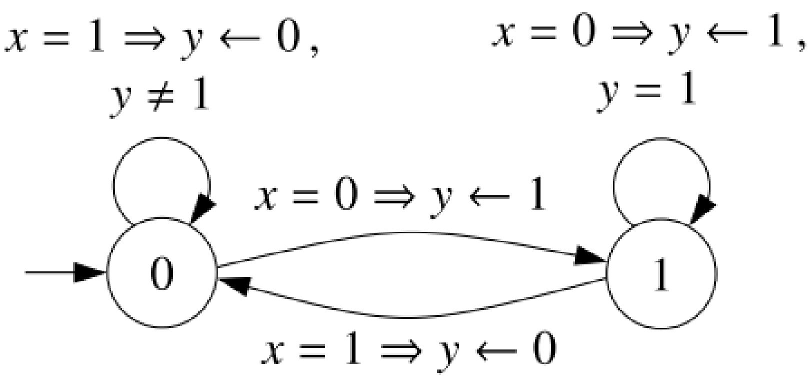
- Therefore: assignment redundant iff none of these potential executions is a real execution of the program.

Networks of automata

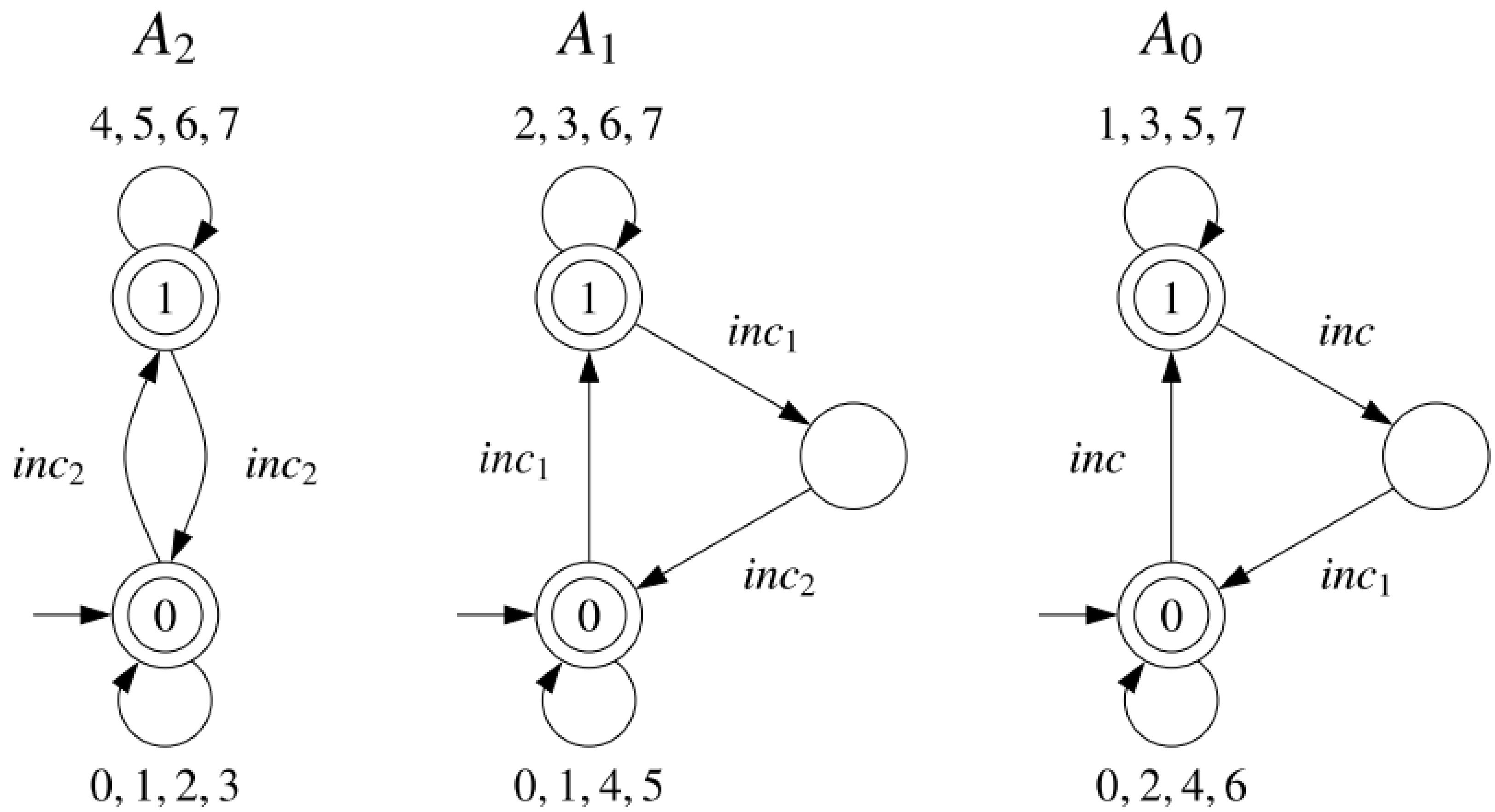
```
while x = 1 do
   if y = 1 then
      x ← 0
   y ← 1 − x
end
```





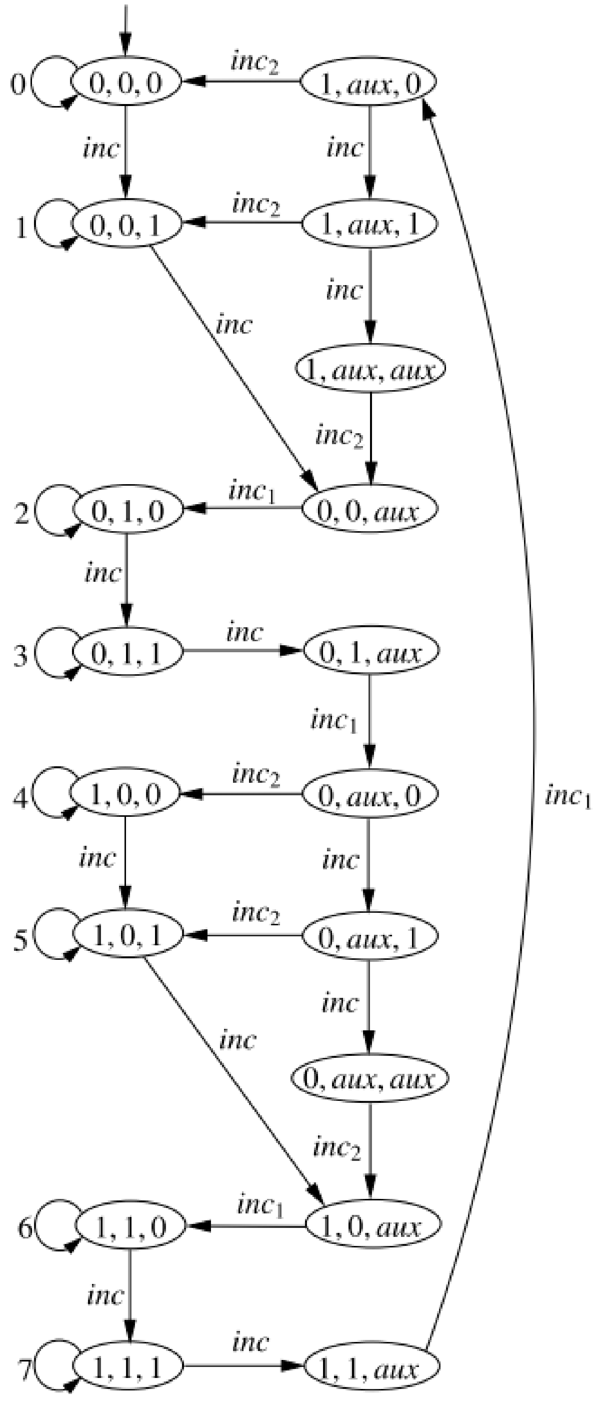


Networks of automata



- Tuple $\mathcal{A} = \langle A_1, \dots, A_n \rangle$ of NFAs .
- Each NFA has its own alphabet Σ_i of actions
- Alphabets usually not disjoint!
- A_i participates in action a if $a \in \Sigma_i$.
- A configuration is a tuple $\langle q_1, \dots, q_n \rangle$ of states, one for each automaton of the network.
- $\langle q_1, \dots, q_n \rangle$ enables a if every participant in a is in a state from which an a-transition is possible.
- Enabled actions can occur, and their occurrence simultaneously changes the states of their participants. Non-participants stay idle and don‘t change their states.

Configuration graph of the network



Asynchronous product

AsyncProduct$(A_1, \ldots, A_n)$

Input: a network of automata $\mathcal{A} = \langle A_1, \ldots, A_n \rangle$, where $A_i = (Q_i, \Sigma_i, \delta_i, Q_{0i}, F_i)$ for every $i = 1, \ldots n$.

Output: NFA $A_1 \otimes \cdots \otimes A_n = (Q, \Sigma, \delta, Q_0, F)$ recognizing $L(\mathcal{A})$.

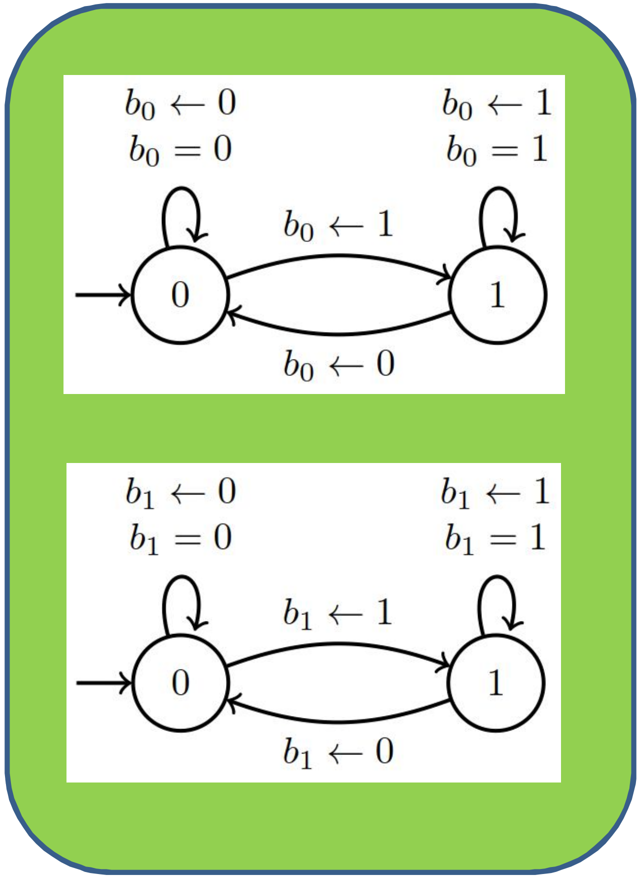
$Q, \delta, F \leftarrow \emptyset$
$Q_0 \leftarrow Q_{01} \times \cdots \times Q_{0n}$
$W \leftarrow Q_0$
while $W \neq \emptyset$ **do**
    pick $[q_1, \ldots, q_n]$ **from** W
    add $[q_1, \ldots, q_n]$ **to** Q
    if $\bigwedge_{i=1}^{n} q_i \in F_i$ **then add** $[q_1, \ldots, q_n]$ **to** F
    for all $a \in \Sigma_1 \cup \ldots \cup \Sigma_n$ **do**
        for all $i \in [1..n]$ **do**
            if $a \in \Sigma_i$ **then** $Q'_i \leftarrow \delta_i(q_i, a)$ **else** $Q'_i = \{q_i\}$
        for all $[q'_1, \ldots, q'_n] \in Q'_1 \times \ldots \times Q'_n$ **do**
            if $[q'_1, \ldots, q'_n] \notin Q$ **then add** $[q'_1, \ldots, q'_n]$ **to** W
            add $([q_1, \ldots, q_n], a, [q'_1, \ldots, q'_n])$ **to** δ
return $(Q, \Sigma, \delta, Q_0, F)$

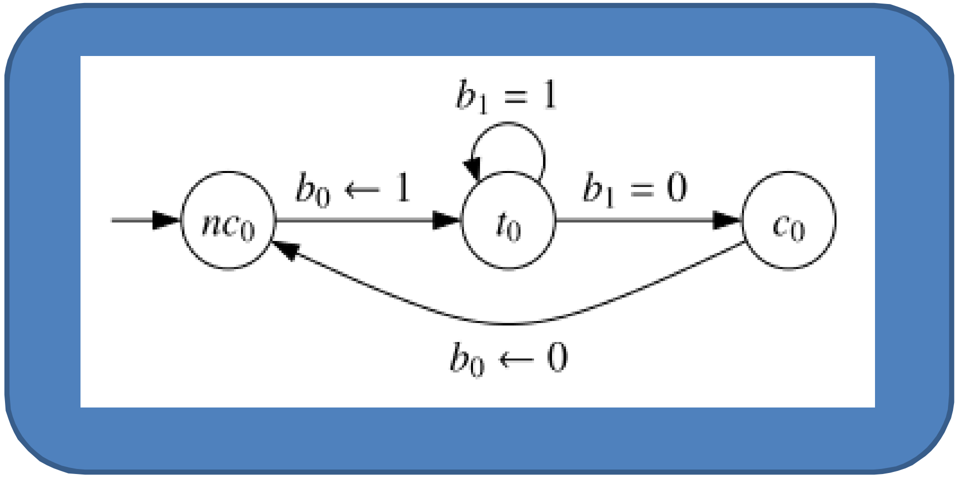
Concurrent programs as networks of automata: Lamport's 1-bit algorithm (JACM86)

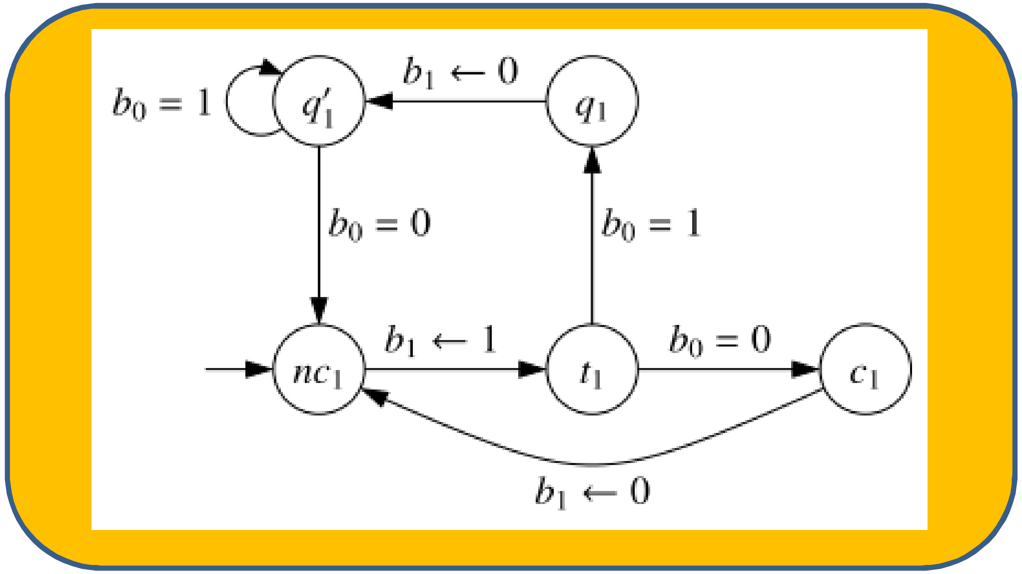
Shared variables: b[0], ..., b[n-1] ∈ {0,1}, initially 0
Process i ∈ {0, ...,n-1}

```
repeat forever
        noncritical section
    T:  b[i]:=1
        for j ∈ {0, ...,i-1}
            if b[j]=1 then  b[i]:=0
                            await ¬b[j]
                            goto T
        for j ∈ {i+1, ...,n-1}  await ¬b[j]
        critical section
        b[i]:=0
```

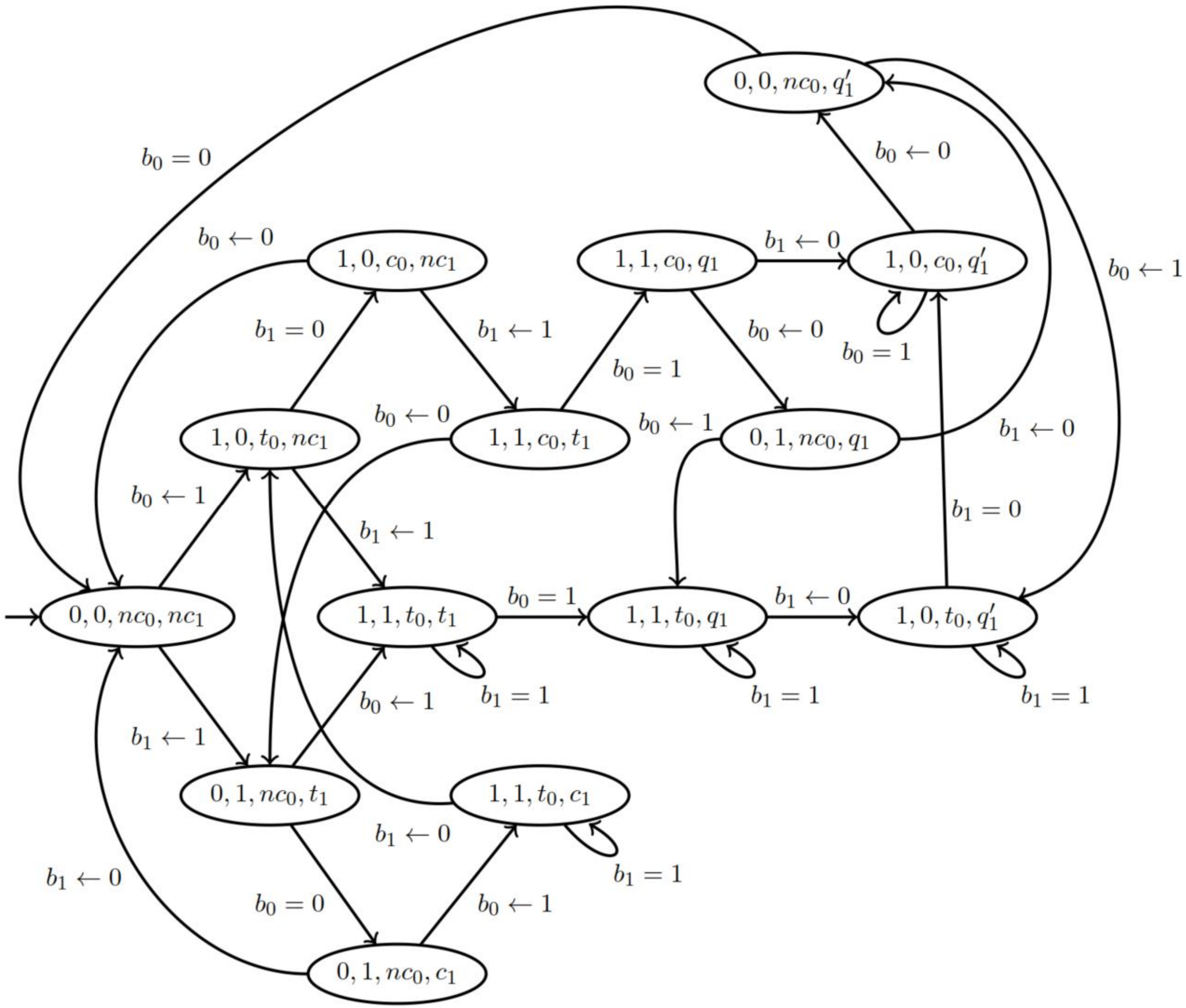
Network for the two-process case







Asynchronous product



Checking properties of the algorithm

- Deadlock freedom: every configuration has at least one successor.
- Mutual exclusion: no configuration of the form $[b_0, b_1, c_0, c_1]$ is reachable
- Bounded overtaking (for process 0): after process 0 signals interest in accessing the critical section, process 1 can enter the critical section at most once before process 0 enters.
  - Let NC_i, T_i, C_i be the configurations in which process i is non-critical, trying, or critical
  - Set of potential executions violating the property:

$$\Sigma^* \; T_0 \; (\Sigma \setminus C_0)^* \; C_1 \; (\Sigma \setminus C_0)^* \; NC_1 \; (\Sigma \setminus C_0)^* \; C_1 \; \Sigma^*$$

The state-explosion problem

- In sequential programs, the number of reachable configurations grows exponentially in the number of variables.
- Proposition: The following problem is PSPACE-complete.
  - Given: a boolean program π (program with only boolean variables), and a NFA A_V recognizing a set of potential executions
  - Decide: Is $E_\pi \cap L(A_V)$ empty?

The state-explosion problem

- In concurrent programs, the number of reachable configurations also grows exponentially in the number of components.
- Proposition: The following problem is PSPACE-complete.
  - Given: a network of automata $\mathcal{A} = \langle A_1, \ldots, A_n \rangle$ and a NFA A_V recognizing a set of potential executions of $\mathcal{A}$
  - Decide: Is $L(A_1 \otimes \cdots \otimes A_n \otimes A_V) = \emptyset$?

On-the-fly Verification

$CheckViol(A_1, \ldots, A_n, V)$

Input: a network $\mathcal{A} = \langle A_1, \ldots A_n \rangle$, where $A_i = (Q_i, \Sigma_i, \delta_i, Q_{0i}, F_i)$ for $1 \leq i \leq n$;
an NFA $V = (Q_V, \Sigma_V, \delta_V, Q_{0v}, F_v)$.

Output: **true** if $L(A_1 \otimes \cdots \otimes A_n \otimes V)$ is nonempty, **false** otherwise.

$Q \leftarrow \emptyset$; $Q_0 \leftarrow Q_{01} \times \cdots \times Q_{0n} \times Q_{0v}$
$W \leftarrow Q_0$
while $W \neq \emptyset$ **do**
  pick $[q_1, \ldots, q_n, q]$ **from** W
  add $[q_1, \ldots, q_n, q]$ **to** Q
  for all $a \in \Sigma_1 \cup \ldots \cup \Sigma_n$ **do**
    for all $i \in [1..n]$ **do**
      if $a \in \Sigma_i$ **then** $Q'_i \leftarrow \delta_i(q_i, a)$ **else** $Q'_i = \{q_i\}$
    $Q' \leftarrow \delta_V(q, a)$
    for all $[q'_1, \ldots, q'_n, q'] \in Q'_1 \times \ldots \times Q'_n \times Q'$ **do**
      if $\bigwedge_{i=1}^{n} q'_i \in F_i$ **and** $q' \in F_v$ **then return true**
      if $[q'_1, \ldots, q'_n, q'] \notin Q$ **then add** $[q'_1, \ldots, q'_n, q']$ **to** W
return false

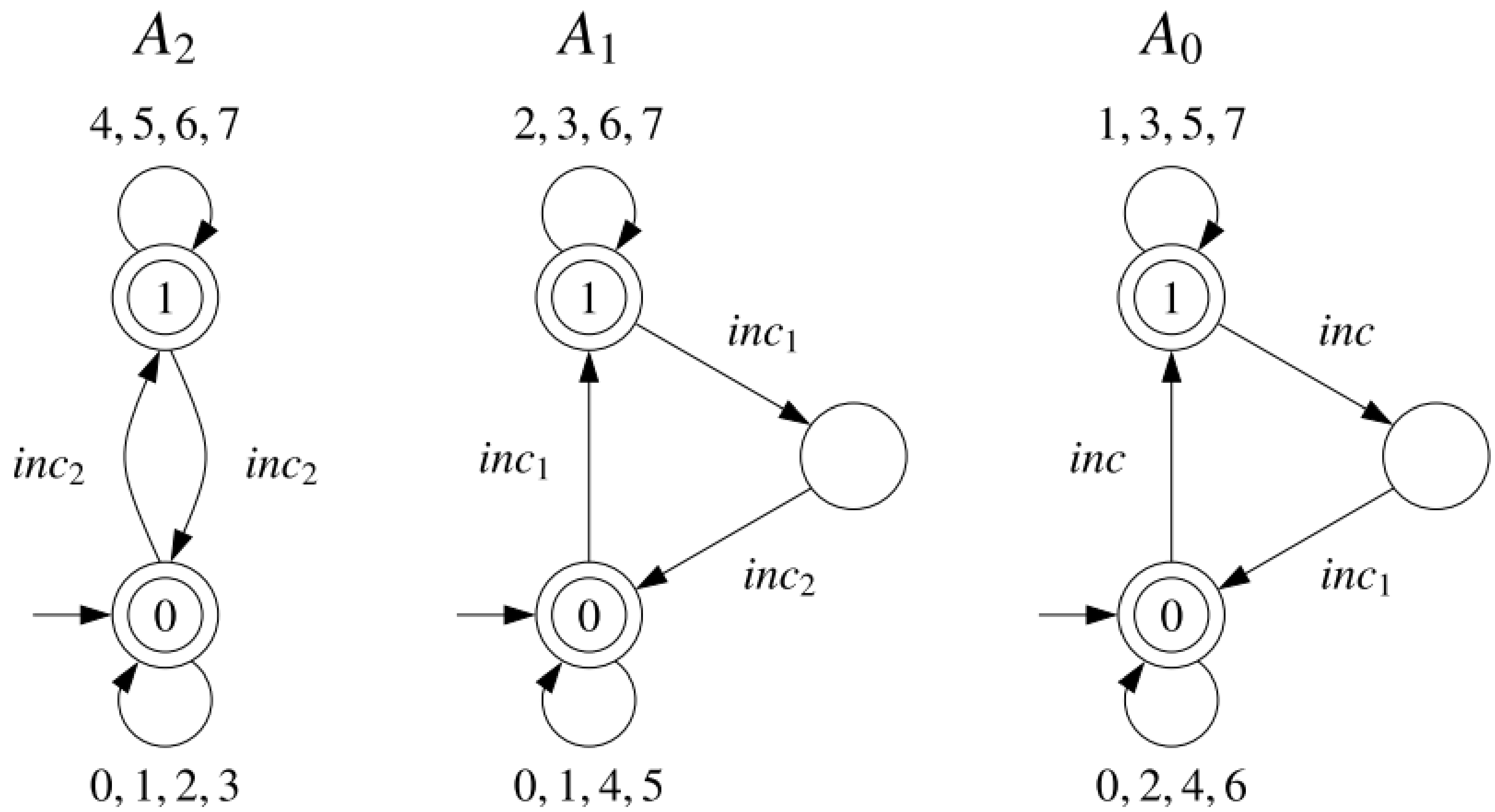
Compositional verification

To check emptiness of an asynchronous product $A_1 \otimes \cdots \otimes A_n$ we can

- Replace A_1 by an automaton A_1' recognizing $proj_{\Sigma \setminus \Sigma_1}(L(A_1))$ and compute $A_{12} = A_1' \otimes A_2$;
- Replace A_{12} by an automaton A_{12}' recognizing $proj_{\Sigma \setminus (\Sigma_1 \cup \Sigma_2)}(L(A_{12}))$ and compute $A_{13} = A_{12}' \otimes A_3$;
- $\cdots$
- Replace $A_{1(n-1)}$ by an automaton $A_{1(n-1)}'$ recognizing $proj_{\Sigma \setminus (\Sigma_1 \cup \cdots \cup \Sigma_{n-1})}\left(L\left(A_{1(n-1)}\right)\right)$ and compute $A_{1n} = A_{1(n-1)}' \otimes A_n$

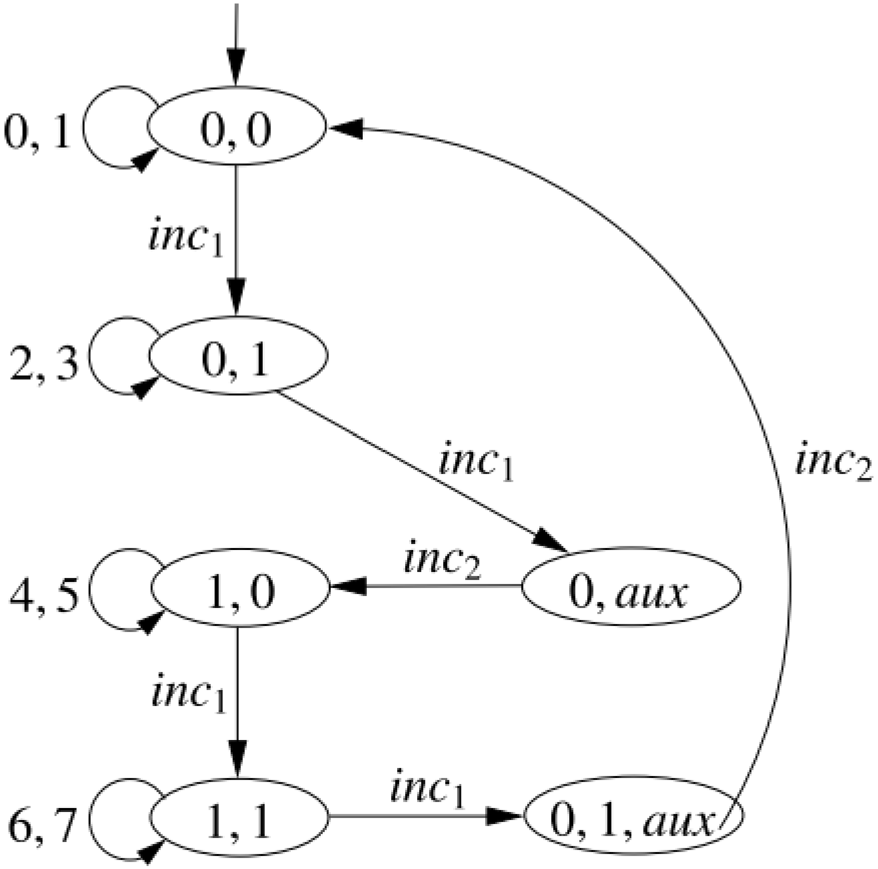
This can save space w.r.t. the direct computation .

Compositional verification

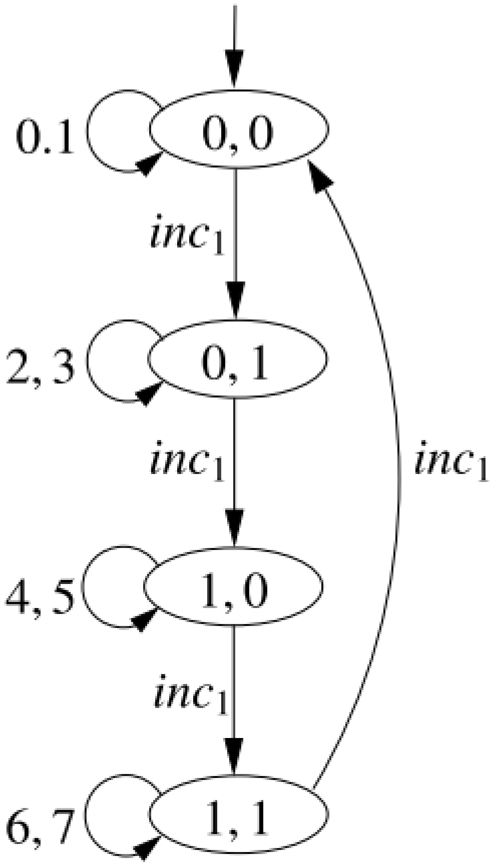


Compositional verification

A_{21}

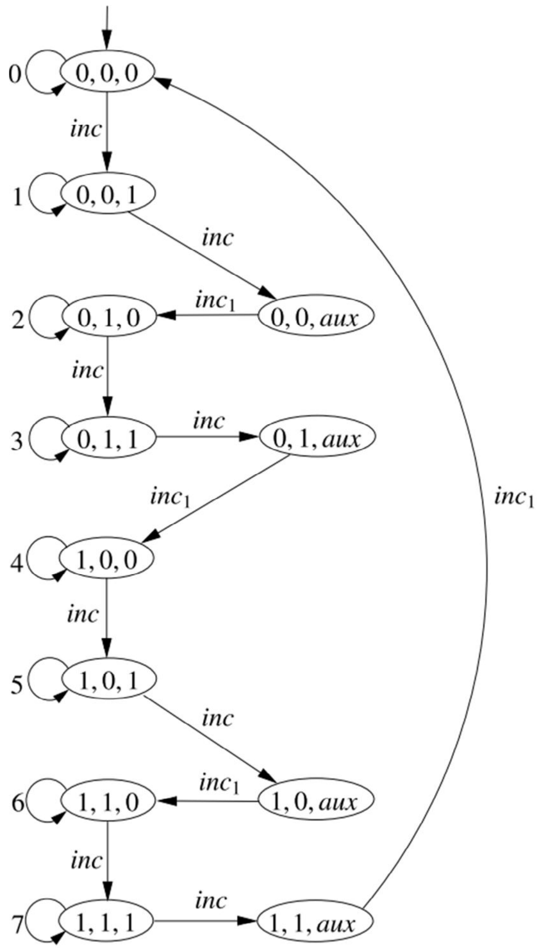


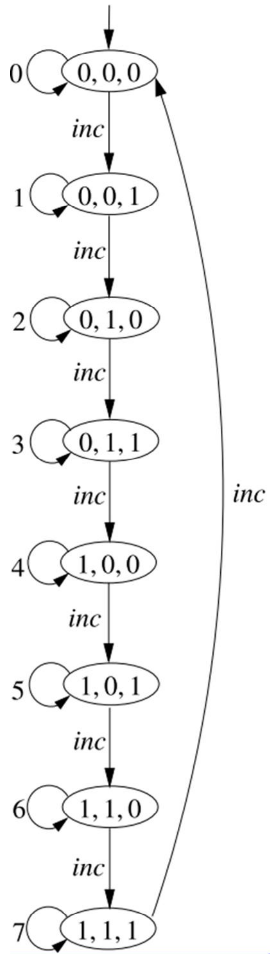
A'_{21}



Compositional verification

A_{20} A'_{20} (proj. on visible actions)



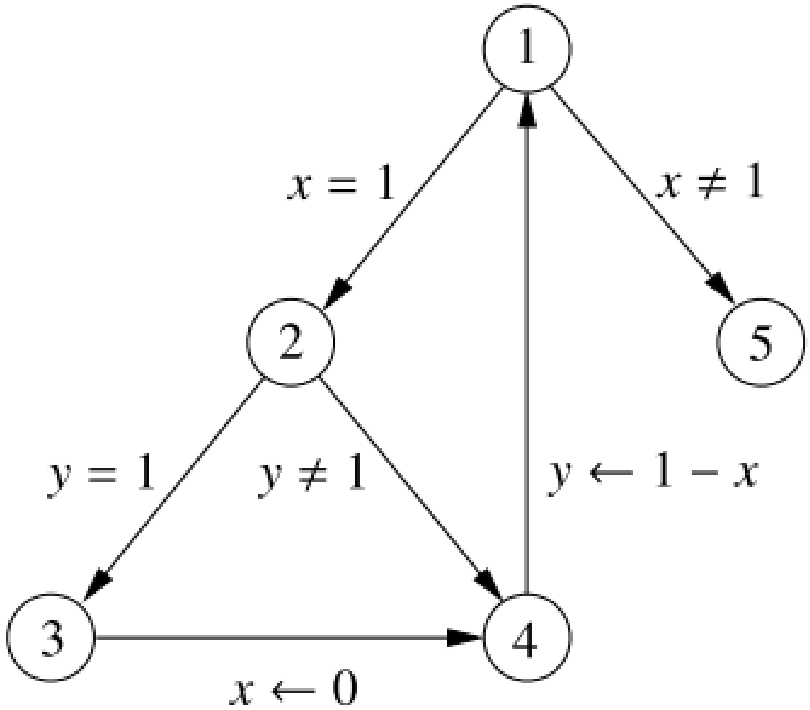


Symbolic exploration

- A technique to palliate the state-explosion problem
- Configurations can be encoded as words.
- The set of reachable configurations of a program can be encoded as a language.
- We use automata to compactly store the set of reachable configurations.

Flowgraphs

```
while x = 1 do
    if y = 1 then
        x ← 0
    y ← 1 − x
end
```



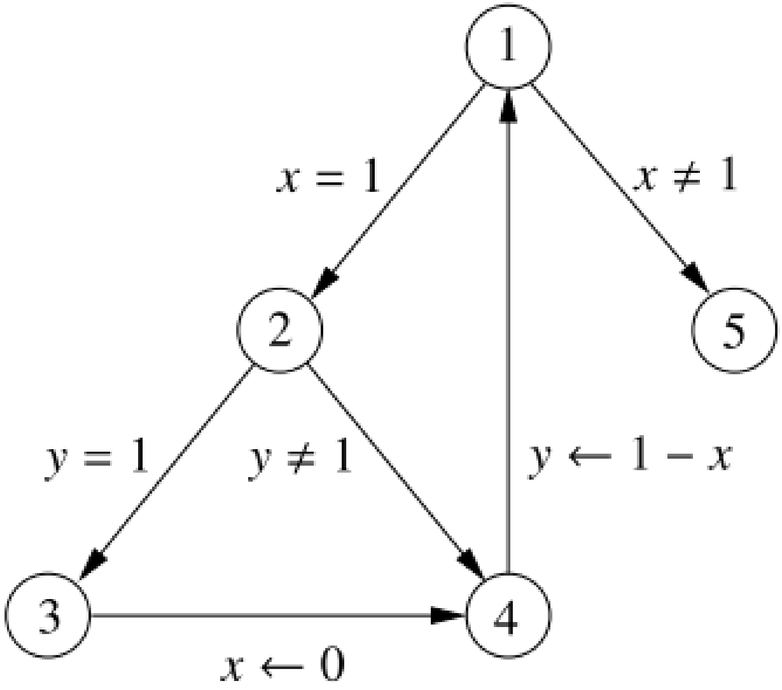
Step relations

- Let l, l' be two control points of a flowgraph.
- The step relation $S_{l,l'}$ contains all pairs

$$([l, x_0, y_0], [l', x_0', y_0'])$$

of configurations such that :

if at point l the current values of x, y are x_0, y_0,
then the program can take a step,
after which the new control point is l', and the new values of x, y are x_0', y_0'.



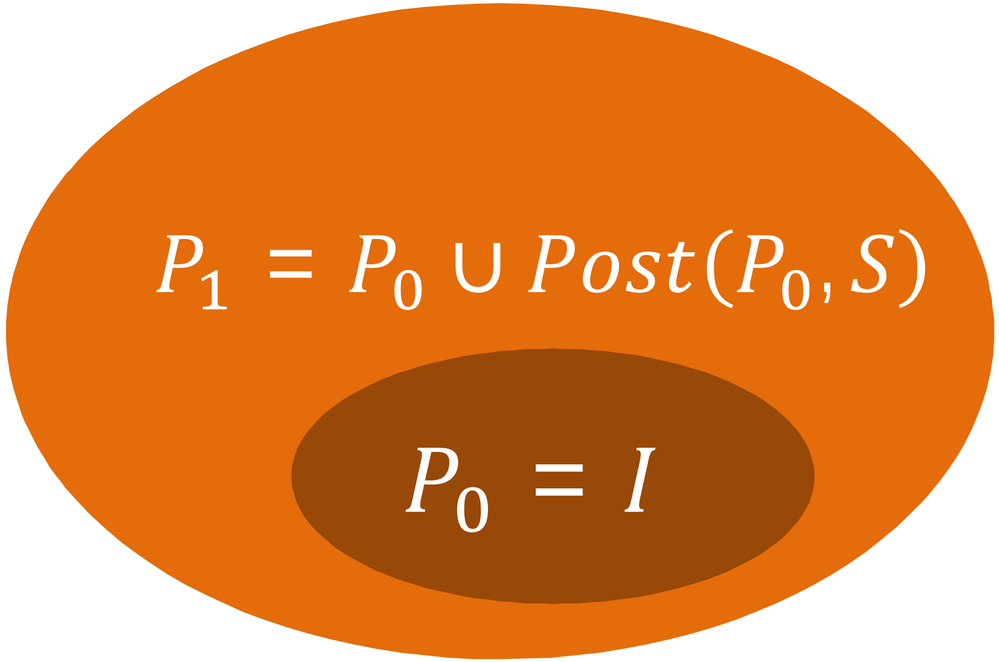
$$S_{4,1} = \{ ([4, x_0, y_0], [1, x_0, 1 - x_0]) \mid x_0, y_0 \in \{0,1\} \}$$

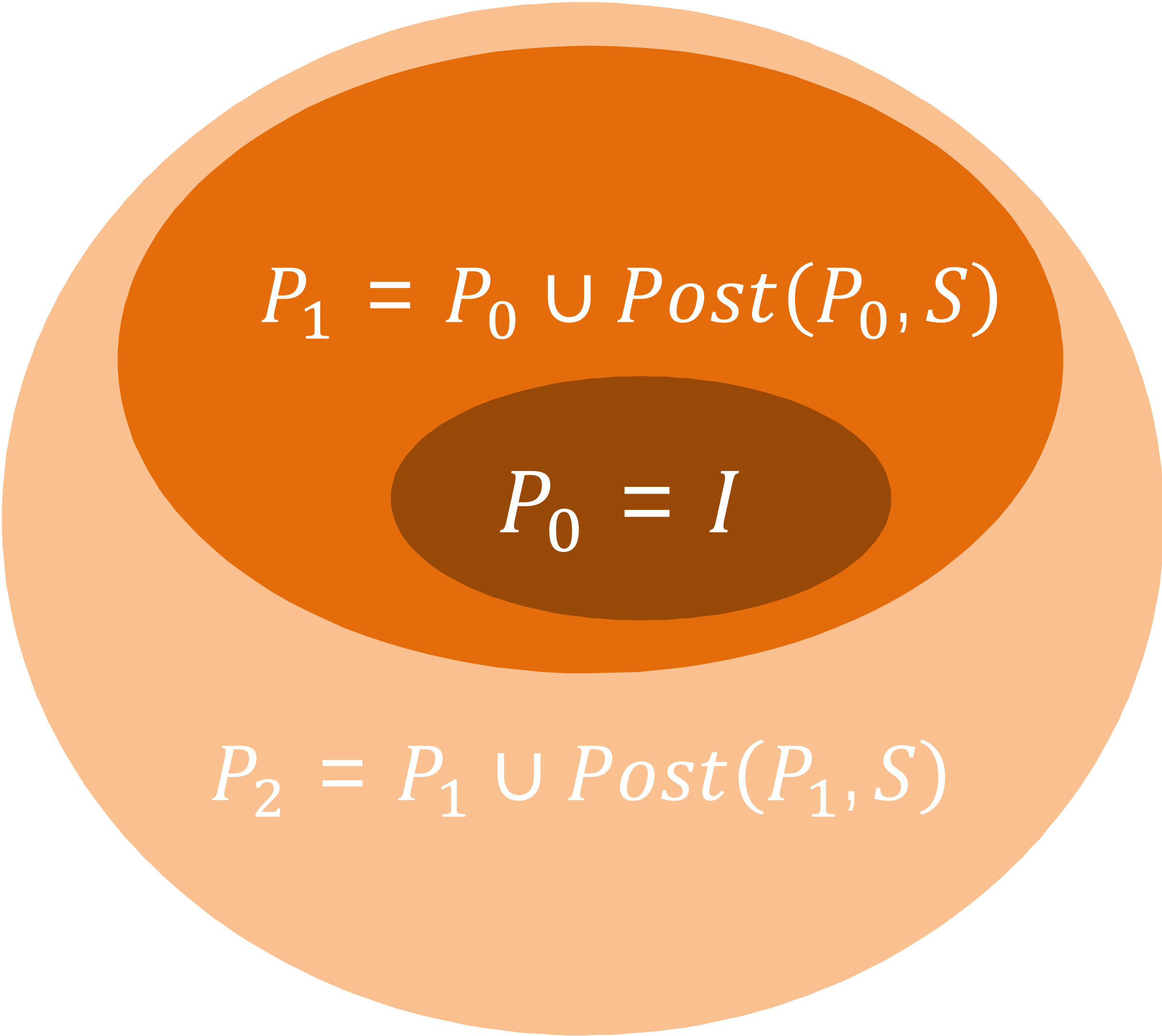
- The global step relation S is the union of the step relations $S_{l,l'}$ for all pairs l, l' of control points.

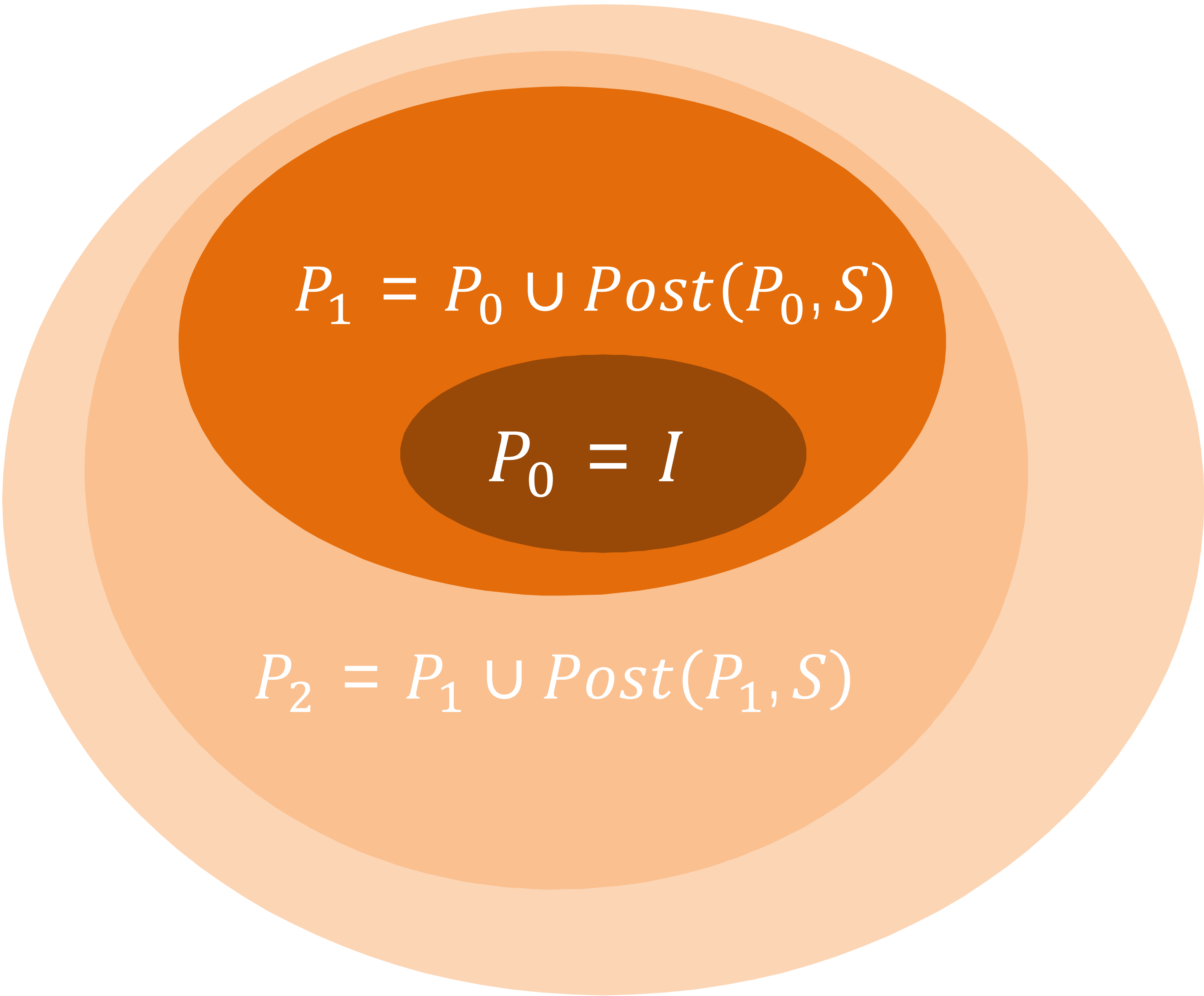
Computing reachable configurations

- Start with the set of initial configurations.
- Iteratively: add the set of successors of the current set of configurations until a fixed point is reached.

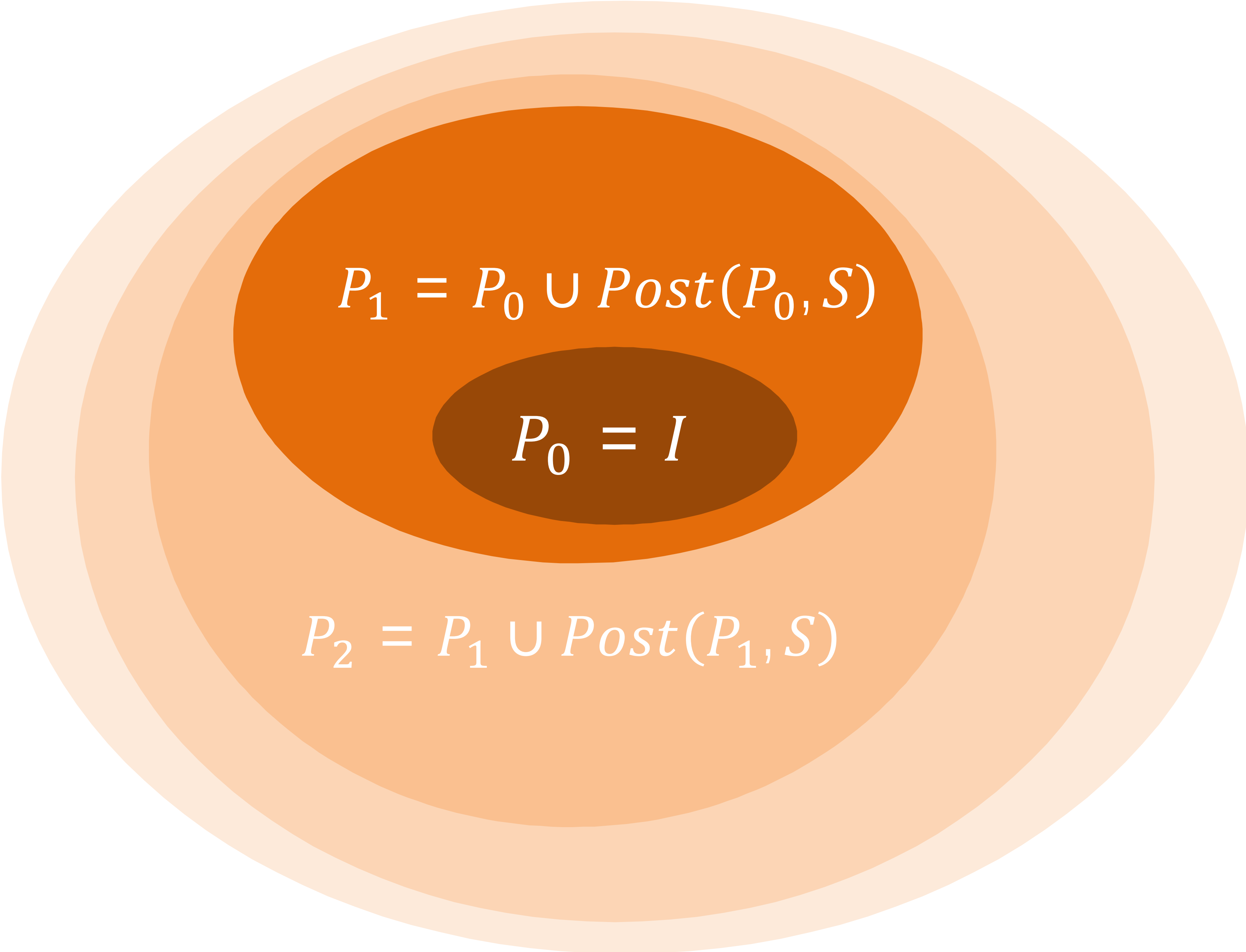
$$P_0 = I$$


$P_1 = P_0 \cup Post(P_0, S)$
$P_0 = I$


$P_1 = P_0 \cup Post(P_0, S)$
$P_0 = I$
$P_2 = P_1 \cup Post(P_1, S)$



$P_1 = P_0 \cup Post(P_0, S)$
$P_0 = I$
$P_2 = P_1 \cup Post(P_1, S)$

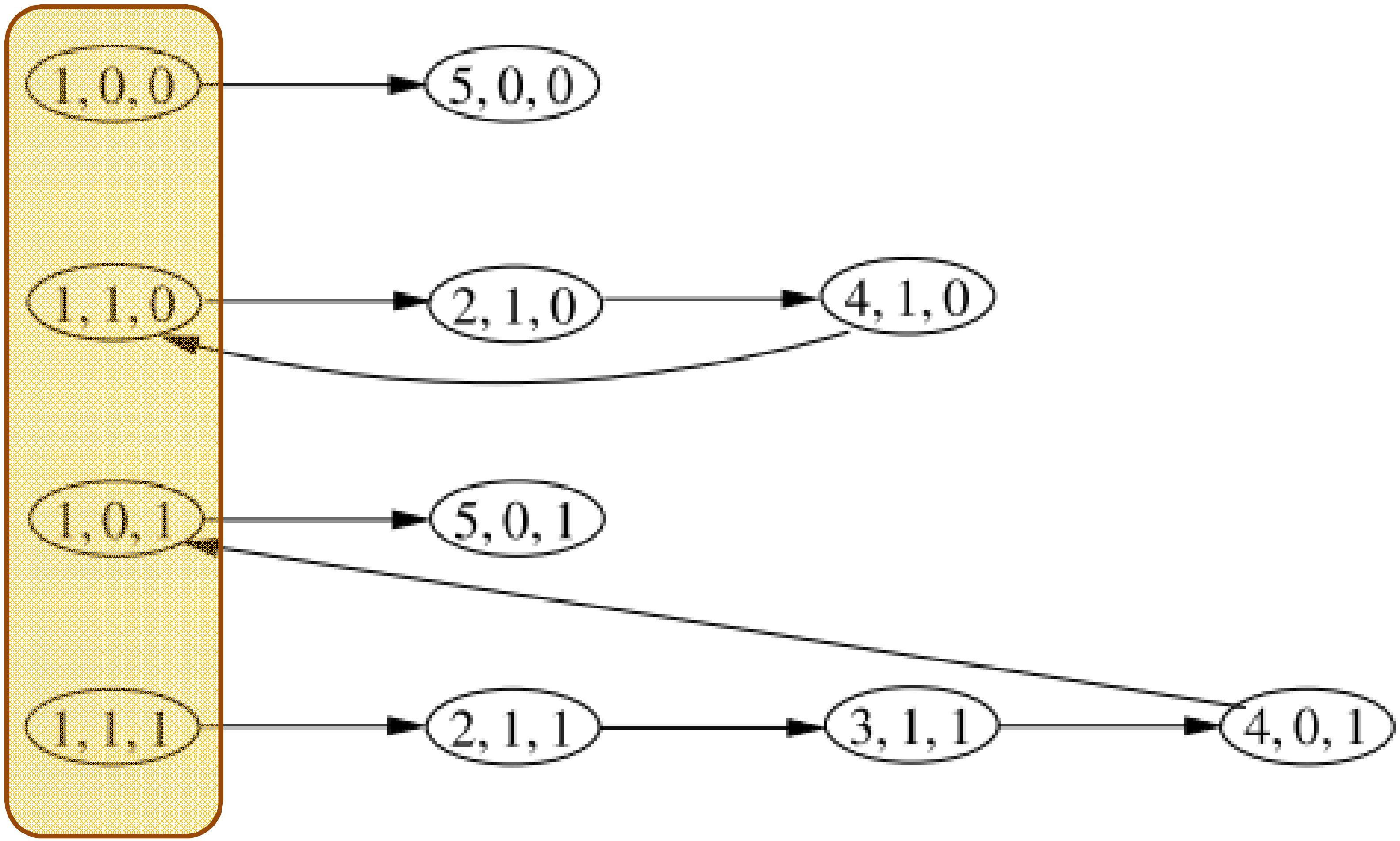

$P_1 = P_0 \cup Post(P_0, S)$
$P_0 = I$
$P_2 = P_1 \cup Post(P_1, S)$

$Reach(I, R)$

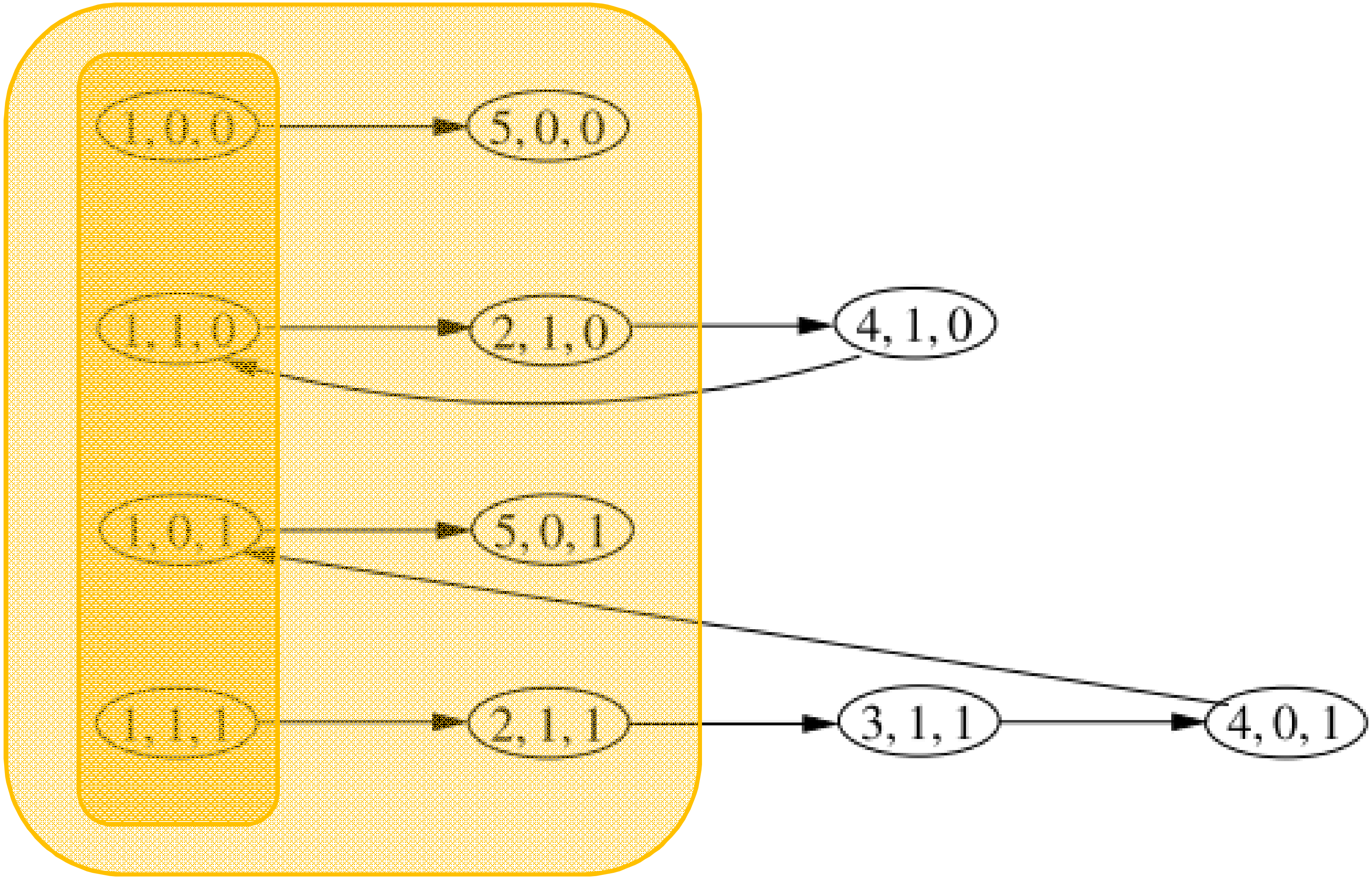
Input: set I of initial configurations; relation R

Output: set of configurations reachable form I

$OldP \leftarrow \emptyset$; $P \leftarrow I$
while $P \neq OldP$ **do**
    $OldP \leftarrow P$
    $P \leftarrow$ **Union**$(P,$ **Post**$(P, S))$
return P



1, 0, 0
5, 0, 0
1, 1, 0
2, 1, 0
4, 1, 0
1, 0, 1
5, 0, 1
1, 1, 1
2, 1, 1
3, 1, 1
4, 0, 1



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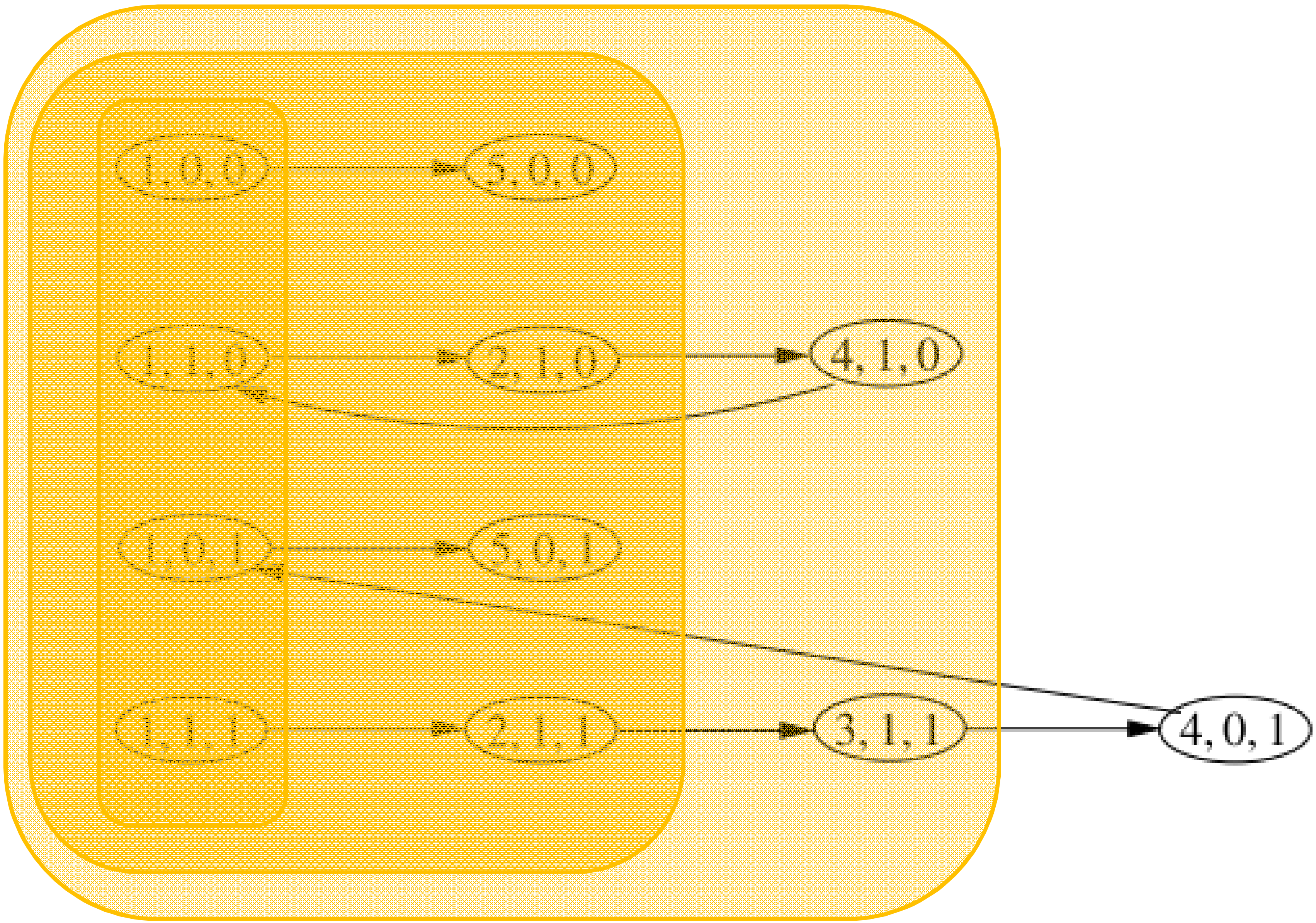
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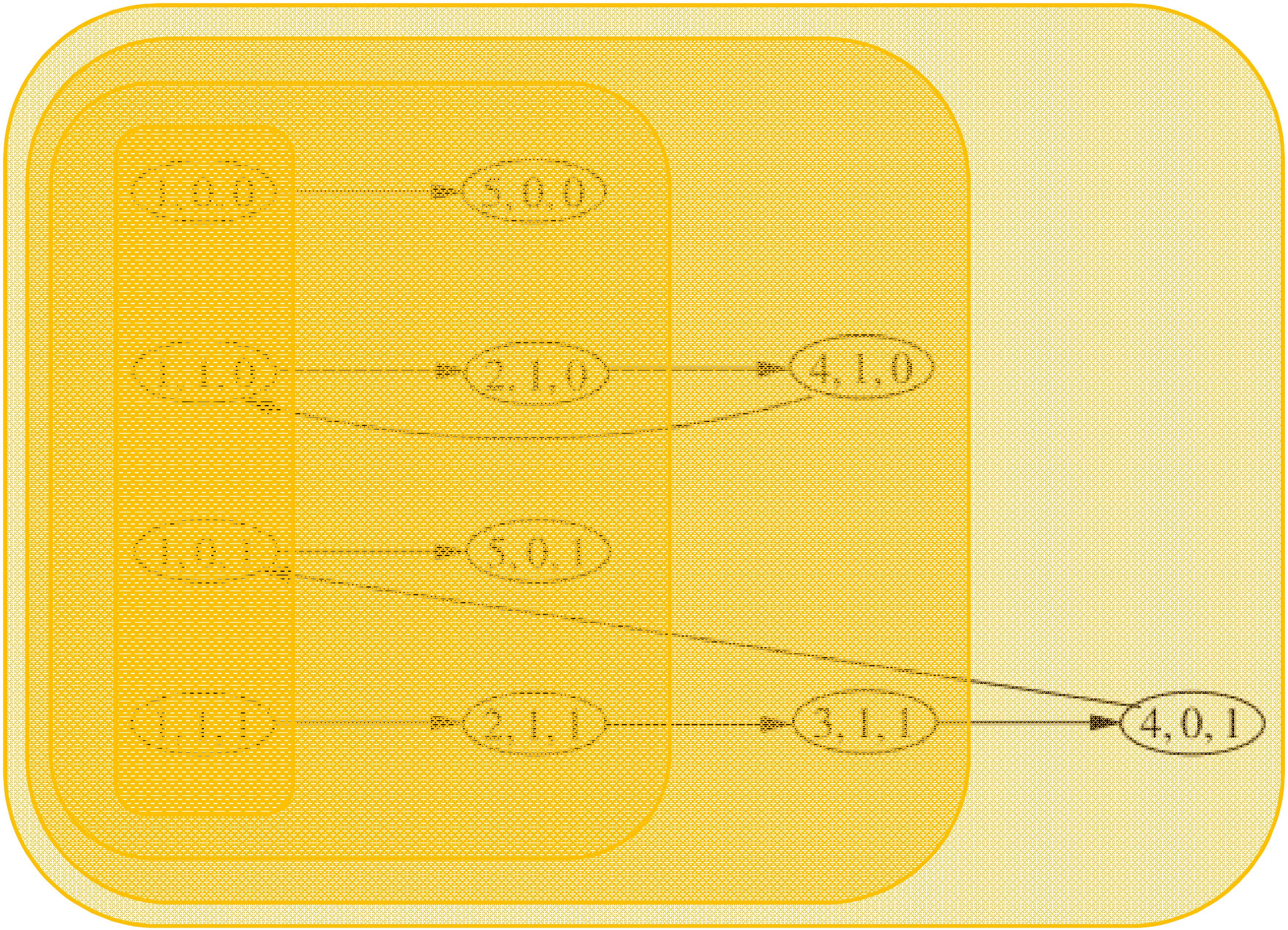
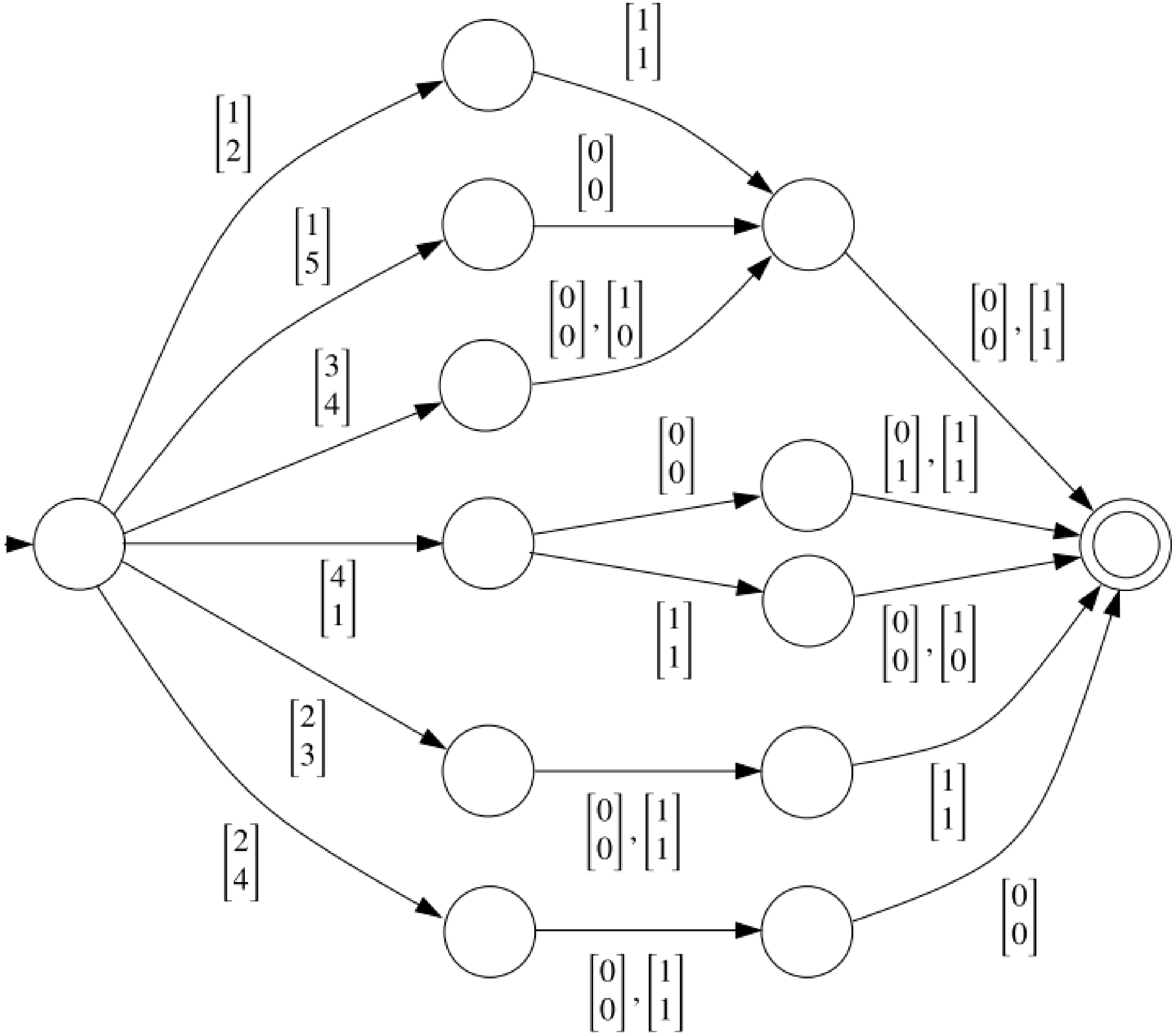
4, 0, 1


5,0,0
2,1,0
4,1,0
5,0,1
2,1,1
3,1,1
4,0,1

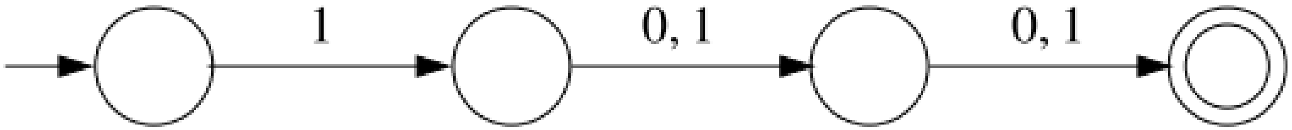
Example: Transducer for the global step relation

```
while x = 1 do
   if y = 1 then
      x ← 0
   y ← 1 − x
end
```

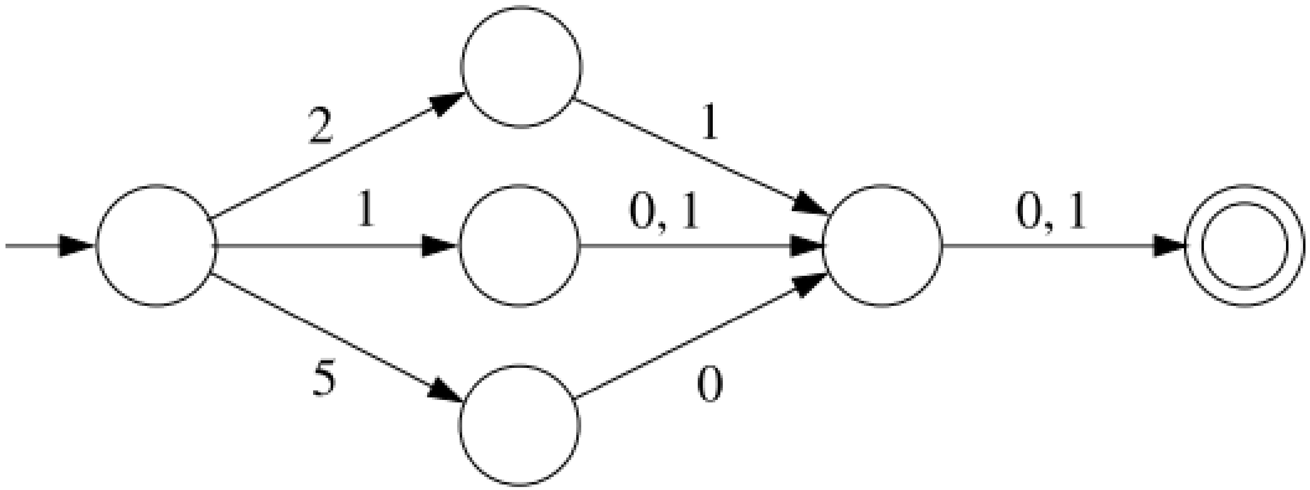


Example: DFAs generated by Reach

- Initial configurations

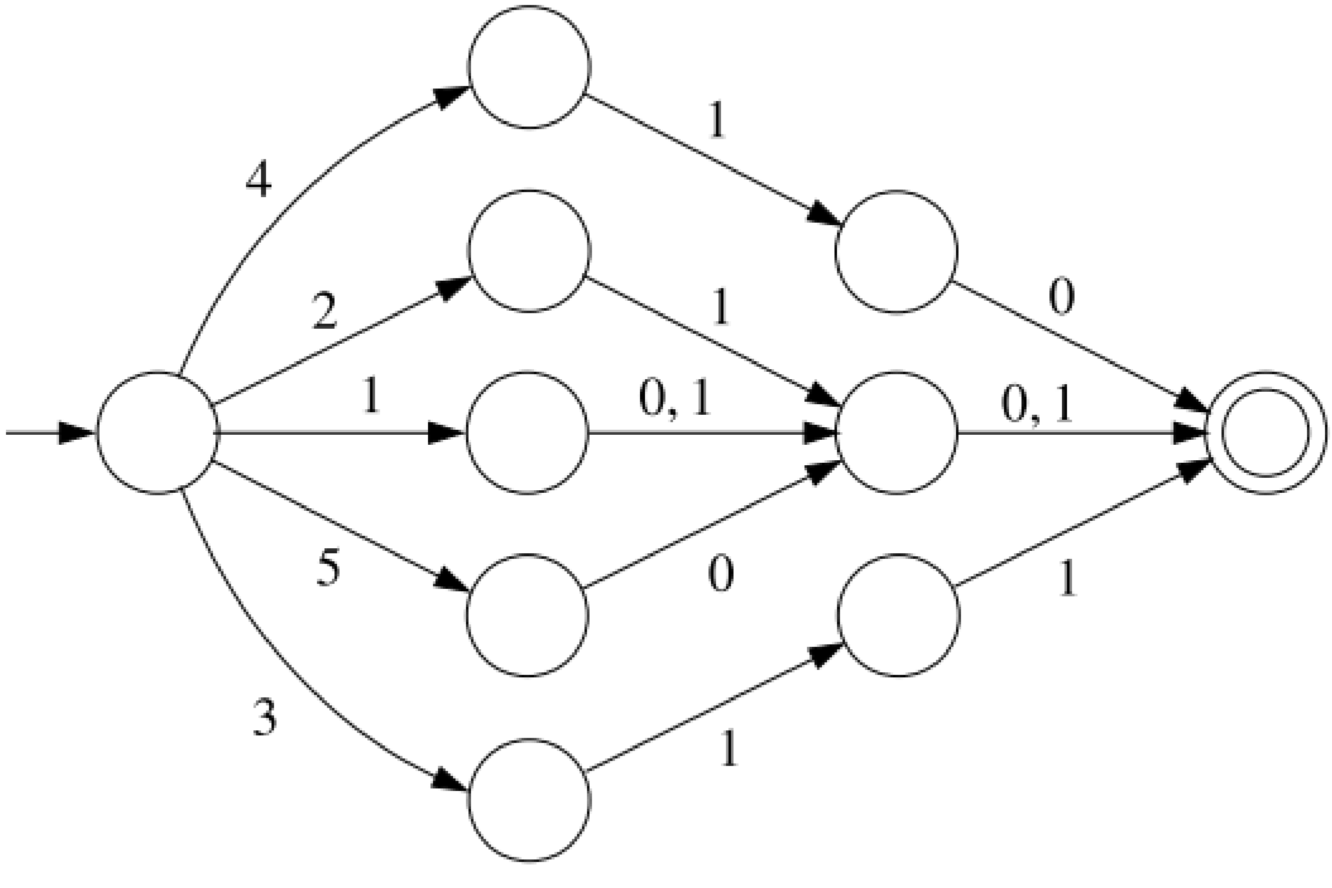


- Configurations reachable in at most 1 step



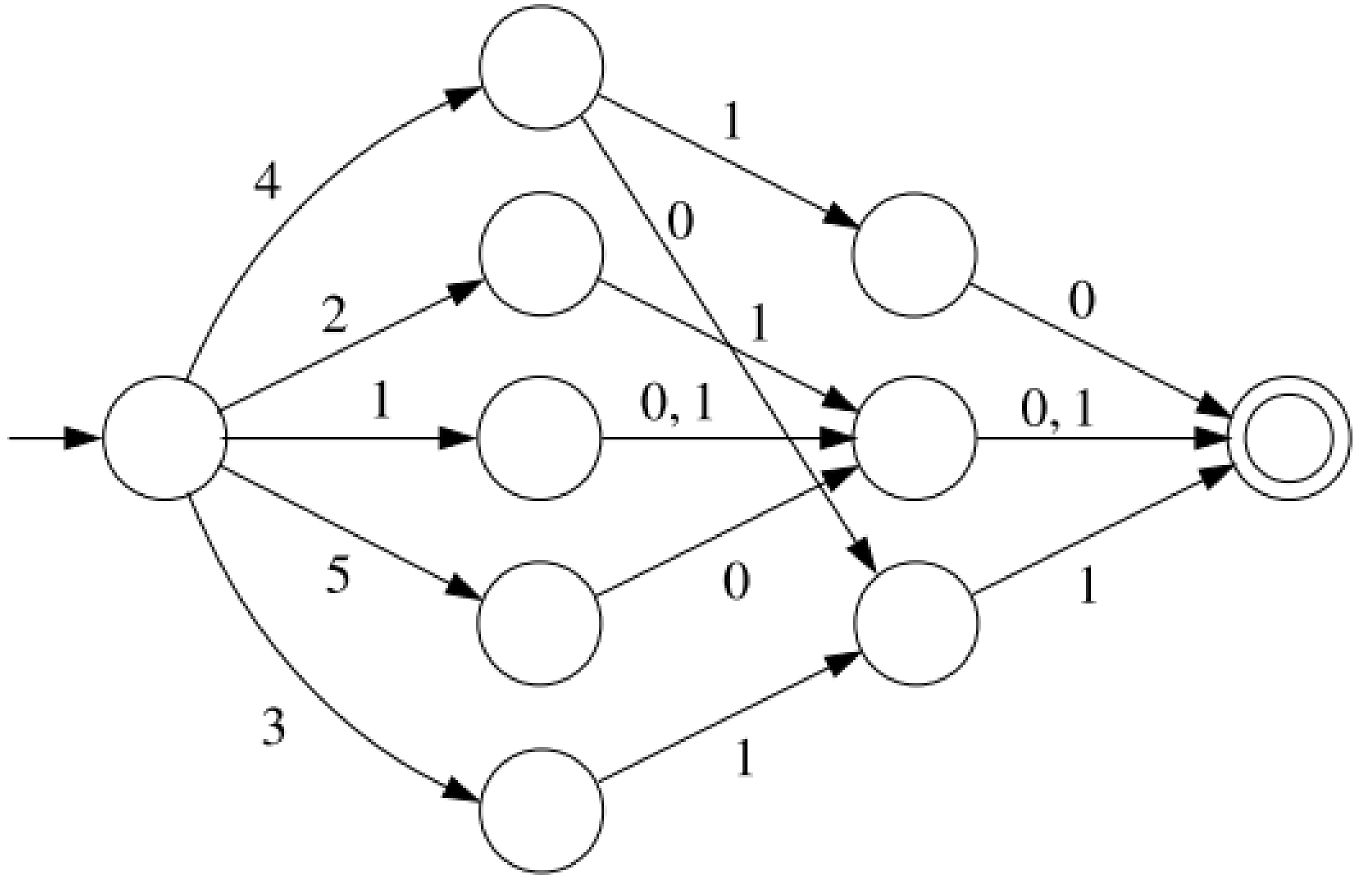
Example: DFAs generated by Reach

- Configurations reachable in at most 2 steps



Example: DFAs generated by Reach

- Configurations reachable in at most 3 steps



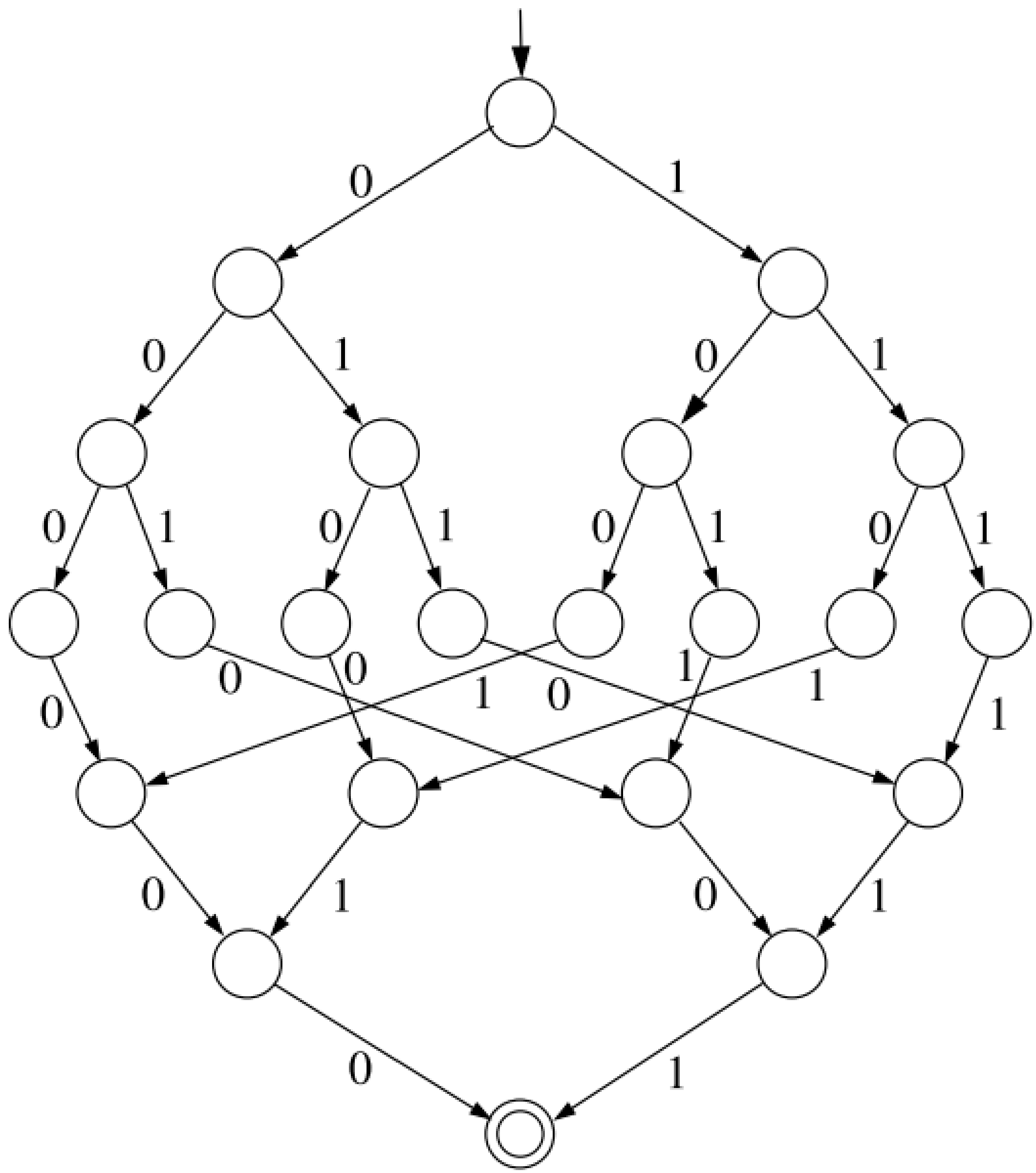
Variable orders

- Consider the set Y of tuples $[x_1, \dots, x_{2k}]$ of booleans such that $x_1 = x_{k+1}, x_2 = x_{k+2}, \dots, x_k = x_{2k}$
- A tuple $[x_1, \dots, x_{2k}]$ can be encoded by the word $x_1 x_2 \dots x_{2k-1} x_{2k}$ but also by the word $x_1 x_{k+1} \dots x_k x_{2k}$.
- For $k = 3$, the encodings of Y are then, respectively

$$\{000000, 001001, 010010, 011011, 100100, 101101, 110110, 111111\}$$

$$\{000000, 000011, 001100, 001111, 110000, 110011, 111100, 111111\}$$

- The minimal DFAs for these languages have very different sizes!



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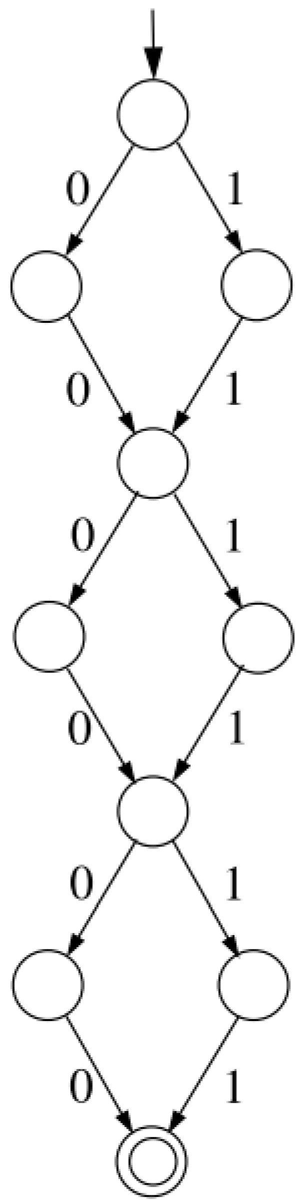

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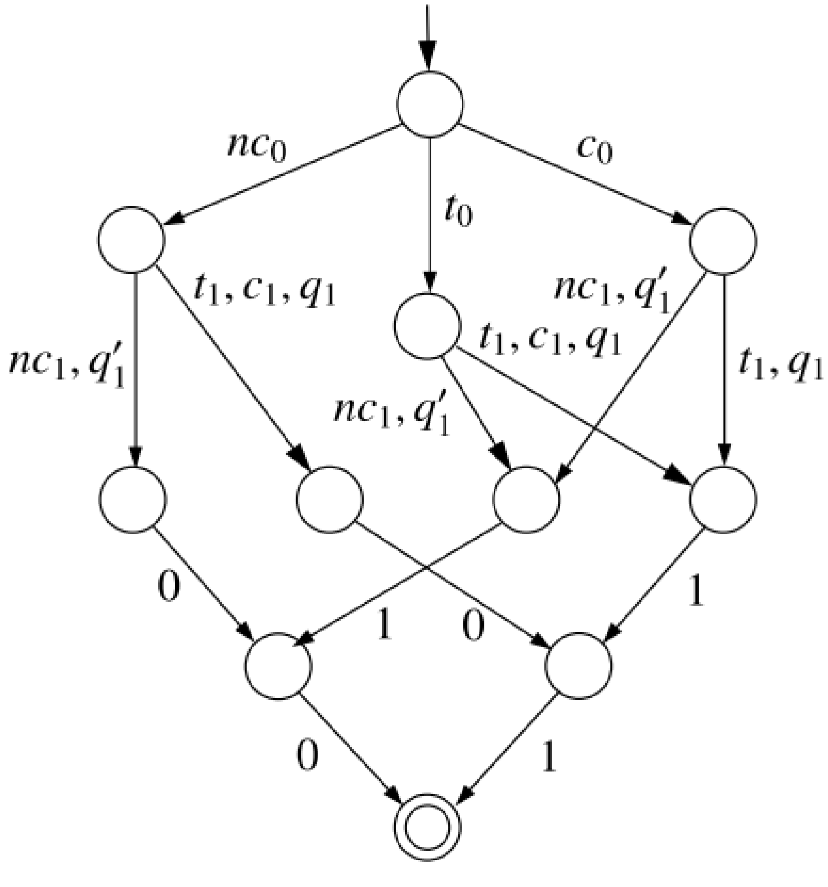
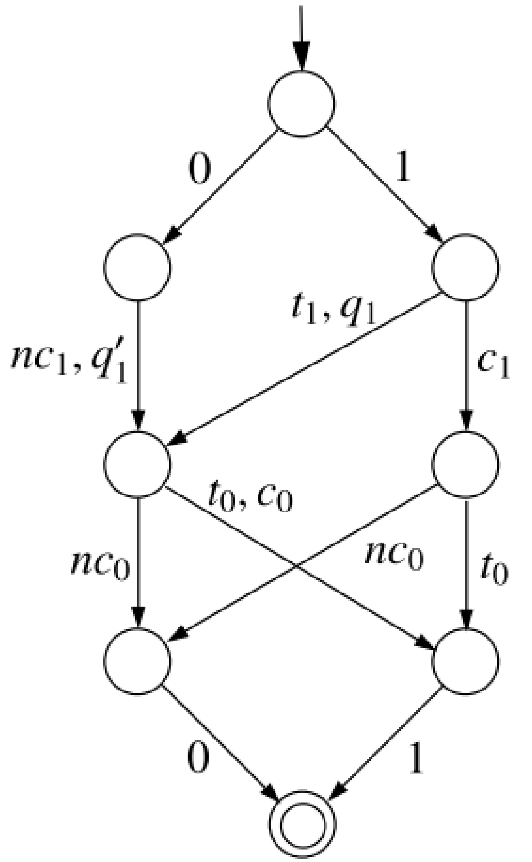
Another example: Lamport‘s algorithm

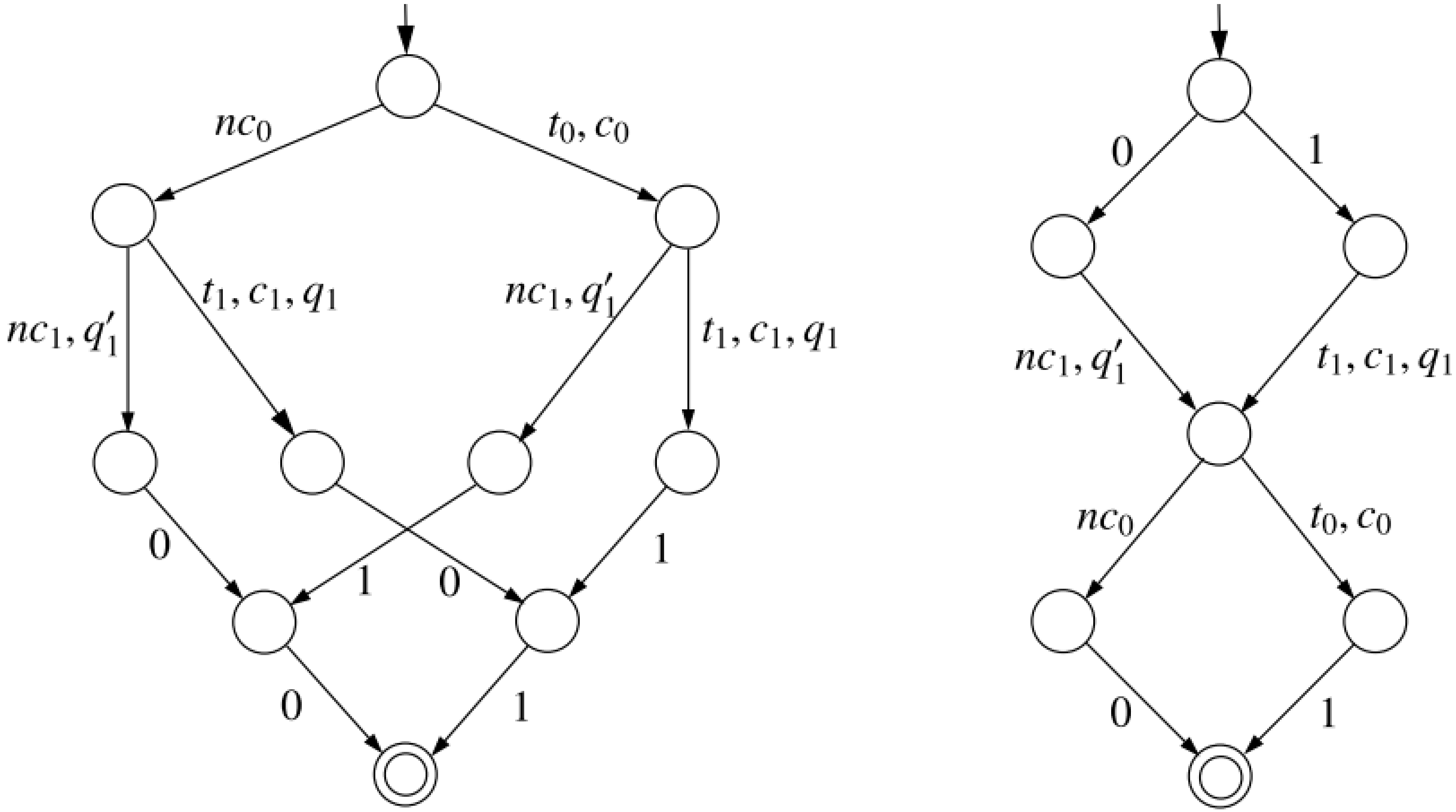


$\langle v_0, v_1, s_0, s_1 \rangle$
encoded by
$s_0 s_1 v_0 v_1$



$\langle v_0, v_1, s_0, s_1 \rangle$
encoded by
$v_1 s_1 s_0 v_0$

Larger sets can yield smaller DFAs!



- DFAs after adding the configuration $\langle c_0, c_1, 1{,}1 \rangle$ to the set

- When encoding configurations, good variable orders can lead to much smaller automata.
- Unfortunately, the problem of finding an optimal encoding for a language represented by a DFA is NP-complete.