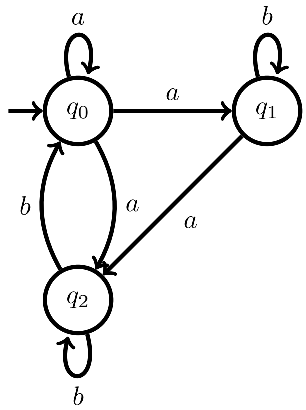
Automata and Formal Languages — Exercise Sheet 10

Exercise 10.1
Give *deterministic* Rabin automata and Muller automata for the following language:

$$L = \{w \in \{a,b\}^\omega : w \text{ contains finitely many } a\text{'s}\}.$$

Exercise 10.2
Give a procedure that translates non-deterministic Rabin automata to non-deterministic Büchi automata.

Exercise 10.3
Consider the following automaton A:



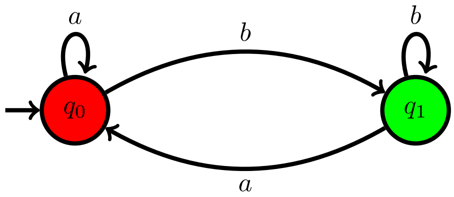
(a) Interpret A as a Rabin automaton with acceptance condition $\{\langle\{q_0, q_2\}, \{q_1\}\rangle\}$. Follow the approach from Exercise 10.2 to construct a Büchi automaton that recognizes the same language as A.

(b) Interpret A as a Muller automaton with acceptance condition $\{\{q_1\}, \{q_0, q_2\}\}$. Use algorithms *NMAtoNGA* and *NGAtoNBA* from the lecture notes to construct a Büchi automaton that recognizes the same language as A.

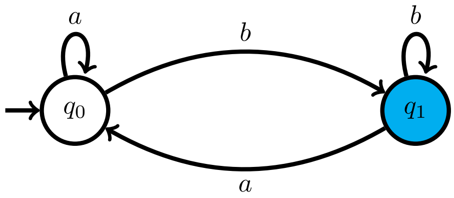
Exercise 10.4
Consider the class of non deterministic automata over infinite words with the following acceptance condition: an infinite run is accepting if it visits a final state *at least once*. Show that no such automaton accepts the language of all words over $\{a, b\}$ containing infinitely many a and infinitely many b.

Solution 10.1

- We give the following Rabin automaton with acceptance condition $\{(\{q_1\}, \{q_0\})\}$, i.e. where q_1 must be visited infinitely often and q_0 must be visited finitely often:



- We give the following Muller automaton with acceptance condition $\{\{q_1\}\}$, i.e. where precisely $\{q_1\}$ must be visited infinitely often:



Solution 10.2

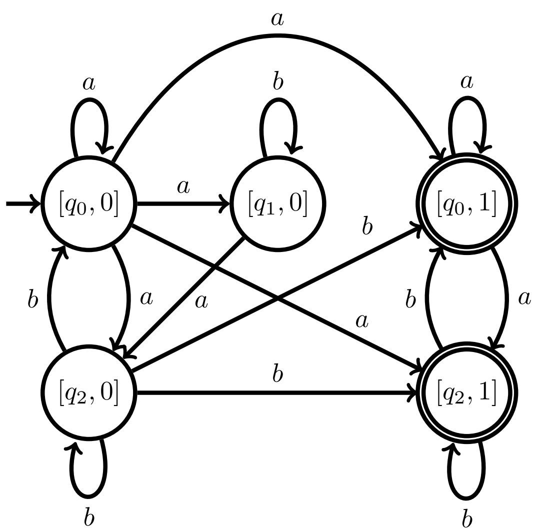
NBA can be easily transformed into nondterministic Rabin automata (NRA) and vice versa, without any exponential blow-up.

NBA → NRA. Just observe that a Büchi condition $\{q_1, \ldots, q_k\}$ is equivalent to the following Rabin condition $\{\, (\{q_1\}, \emptyset), \ldots, (\{q_n\}, \emptyset) \,\}$.

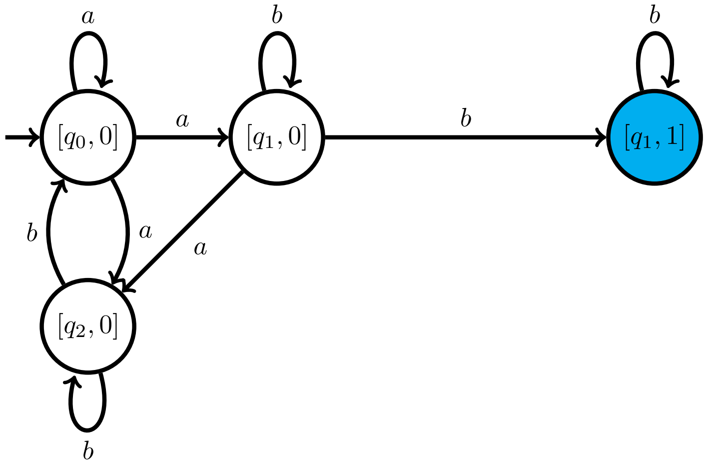
NRA → NBA. Given a Rabin automaton $A = \big(Q, \Sigma, Q_0, \delta, \{\langle F_0, G_0\rangle, \ldots, \langle F_{m-1}, G_{m-1}\rangle\}\big)$, it follows easily that, as in the case of Muller automata, $L_\omega(A) = \bigcup_{i=0}^{m-1} L_\omega(A_i)$ holds for the NRAs $A_i = (Q, \Sigma, Q_0, \delta, \{\langle F_i, G_i\rangle\})$. So it suffices to translate each A_i into an NBA and take the union of the obtained NBAs. Since an accepting run ρ of A_i satisfies $\inf(\rho) \cap G_i = \emptyset$, from some point on ρ only visits states of $Q \setminus G_i$. So ρ consists of an initial *finite* part, say ρ_0, that may visit all states, and an infinite part, say ρ_1, that only visits states of $Q \setminus G_i$. So we take two copies of A_i. Intuitively, A_i' simulates ρ by executing ρ_0 in the first copy, and ρ_1 in the second. The condition that ρ_1 must visit some state of F_i infinitely often is enforced by taking F_i as Büchi condition.

Solution 10.3

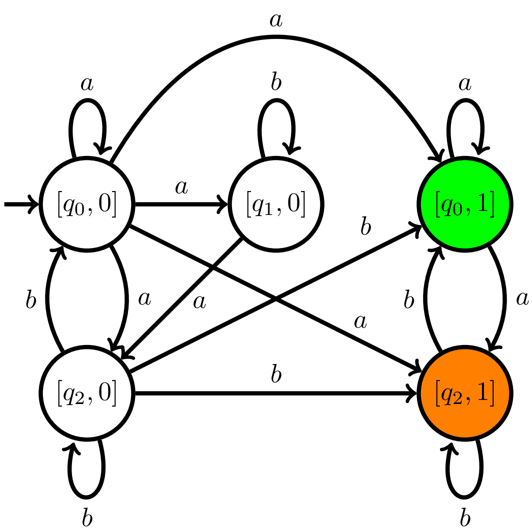
(a)



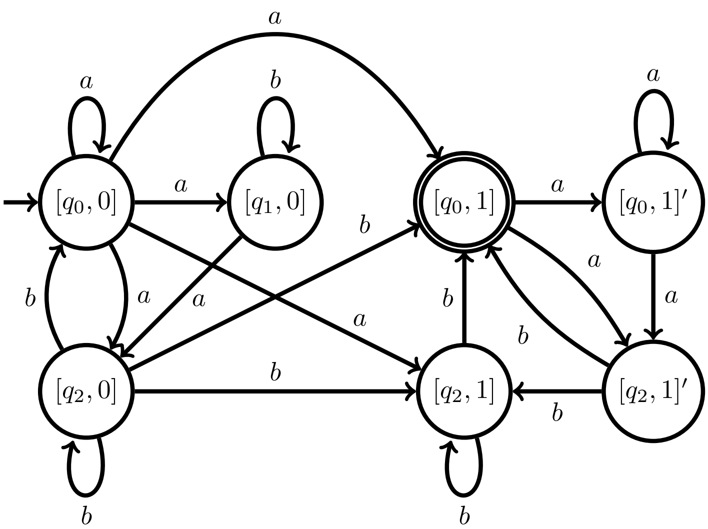
(b) We must first construct two generalized Büchi automata A and B for $\{q_1\}$ and $\{q_0, q_2\}$ respectively. Automaton A is as follows with acceptance condition $\{\{q_1\}\}$:



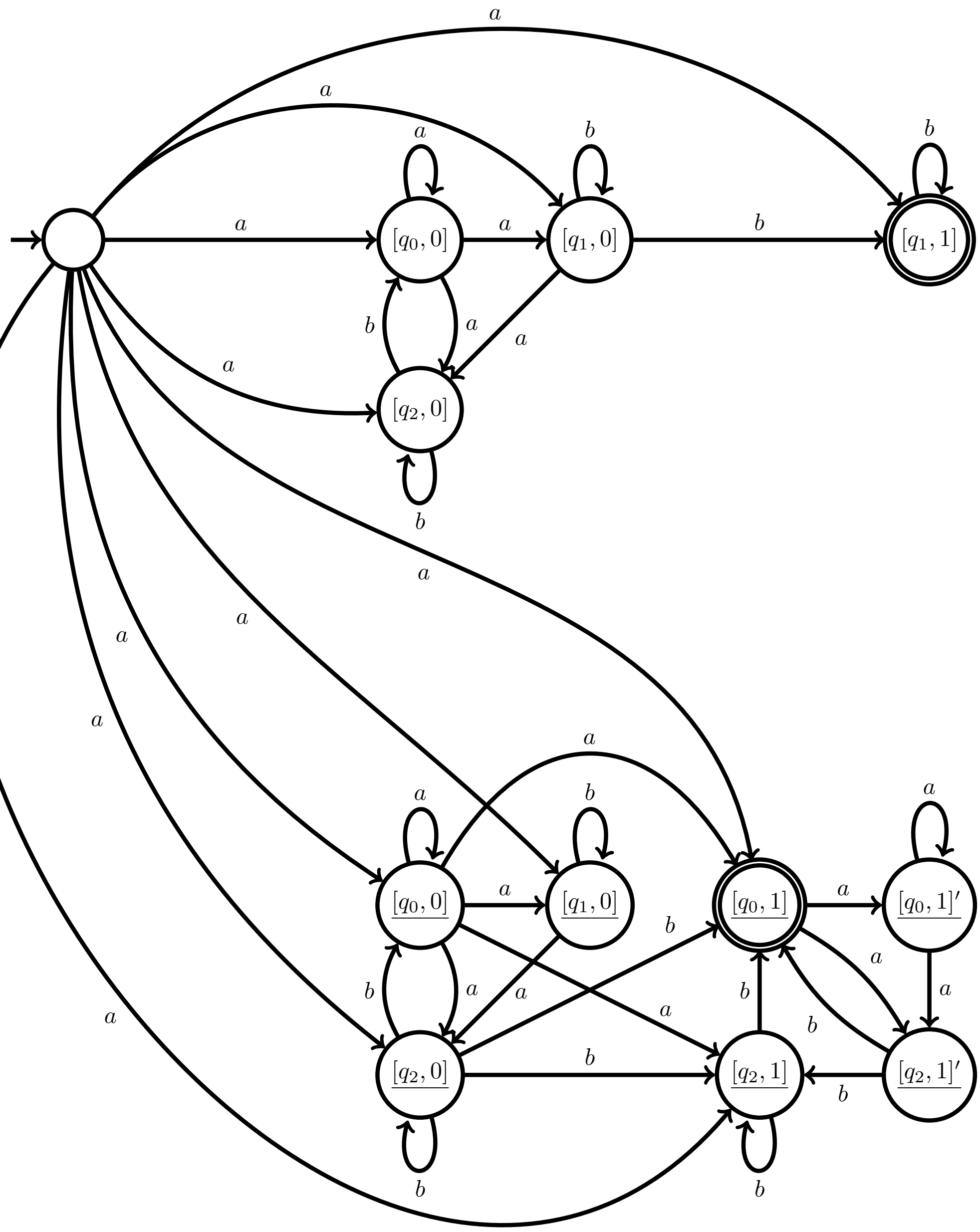
Automaton B is as follows with acceptance condition $\{\{q_0\}, \{q_2\}\}$:



The resulting generalized Büchi automaton is the union of A and B. Note that A is essentially already a standard Büchi automaton, it suffices to make state $[q_1, 1]$ accepting. However, it remains to convert B into a standard Büchi automaton B':



Altogether, we obtain the following Büchi automaton:



★ Since Büchi automata can have multiple initial states, we can also simply take the disjoint union of both automata, i.e. have them side by side instead of adding a single new initial.

Solution 10.4

Suppose there is such an automaton $B = (Q, \{a, b\}, \delta, Q_0, F)$ recognizing L. Since $w = (ab^{|Q|})^\omega$ belongs to L, there exist $u, v \in \{a, b\}^*$, $q_{\text{init}} \in Q_0$, $q_{\text{acc}} \in F$, and $q_0, q_1, \ldots q_{|Q|} \in Q$ such that $uv = (ab^{|Q|})^m a$ for some $m \geq 1$, and

$$q_{\text{init}} \xrightarrow{u} q_{\text{acc}} \xrightarrow{v} q_0 \xrightarrow{b} q_1 \xrightarrow{b} \cdots \xrightarrow{b} q_{|Q|}$$

By the pigeonhole principle, there exist $0 \leq i < j \leq |Q|$ such that $q_i = q_j$. Therefore,

$$q_{\text{init}} \xrightarrow{u} q_{\text{acc}} \xrightarrow{vb^i} q_i \xrightarrow{b^{j-i}} q_j \xrightarrow{b^{j-i}} q_j \xrightarrow{b^{j-i}} \cdots$$

We conclude that $uvb^i(b^{j-i})^\omega$ is accepted by B, which is a contradiction.