Technische Universität München
Winter term 2018/19
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Automata and Formal Languages — Retake Exam

- You have 120 minutes to complete the exam.
- Answers must be written in a separate booklet. Do not answer on the exam.
- Please let us know if you need more paper.
- Write your name and Matrikelnummer on every sheet.
- Write with a non-erasable pen. Do not use red or green.
- You are not allowed to use auxiliary means other than pen and paper.
- You can obtain 40 points. You need 17 points to pass.

Question 1 $((1+1+1+1) + (2+1+1+1) = 9$ **points)**

1. Prove: Every regular language can be recognised by an ϵ-NFA with a single initial state and a single final state.
2. Disprove: Every ω-regular language can be recognised by an NBA with a single initial state and a single accepting state.
3. Prove or disprove: For every regular language L, there exists a *planar* ϵ-NFA N such that $L = L(N)$. Recall that a graph is planar if it can be drawn without edge crossings. Note that self-loops are irrelevant for planarity.
4. Let $\Sigma = \{a, b\}$ be a finite alphabet. Give a sentence of MSO(Σ) for the language $L = a^*b^*$.

Let D be a deterministic automaton and N be a nondeterministic automaton. Let $L(D)$ denote the language of D when we interpret D as a DFA and $L_\omega(D)$ when we interpret D as a DBA. Analogously, let $L(N)$ denote the language of N when we interpret N as a NFA and $L_\omega(N)$ when we interpret N as a NBA.

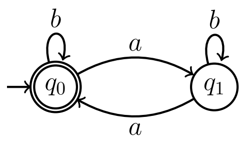
5. Prove: $L(D)$ is infinite if and only if $L_\omega(D)$ is non-empty.
6. Disprove: $L(N)$ is infinite if and only if $L_\omega(N)$ is non-empty.
7. Prove or disprove: The LTL formulas $(\mathbf{G}p)\ \mathbf{U}\ (\mathbf{G}q)$ and $\mathbf{G}(p\ \mathbf{U}\ q)$ are equivalent.
8. Give an ω-regular expression over the alphabet $\Sigma = 2^{\{a,b\}}$ for the formula $\mathbf{FG}a \wedge \mathbf{GF}b$.

Question 2 (2 + 3 = 5 points)
The *perfect shuffle* of two languages $L, L' \in \Sigma^*$ is defined as:

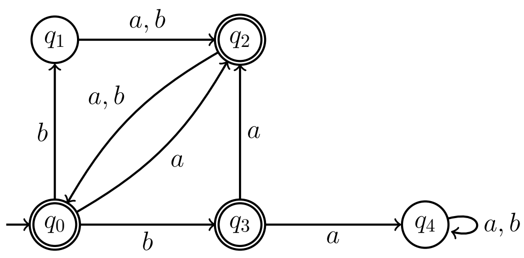
$$L \mathbin{\widetilde{\|}} L' := \{w = a_1 b_1 \cdots a_n b_n \in \Sigma^* : \ a_1 \cdots a_n \in L \wedge b_1 \cdots b_n \in L' \}$$

Notice that $a_1 \cdots a_n$ and $b_1 \cdots b_n$ have to have the same length. For example, if $L_1 = \{\epsilon, a, bb\}$ and $L_2 = \{\epsilon, d, cc, eee\}$ then $L_1 \mathbin{\widetilde{\|}} L_2 = \{\epsilon, ad, bcbc\}$.

1. Give a construction that takes two DFAs A and B as input, and returns a DFA accepting $L(A) \mathbin{\widetilde{\|}} L(B)$.
2. Apply your construction to two copies of the following DFA D and give an interpretation of the states of the resulting DFA:



Question 3 (4 points)
Use the algorithm *UnivNFA* to test whether the following NFA is universal.



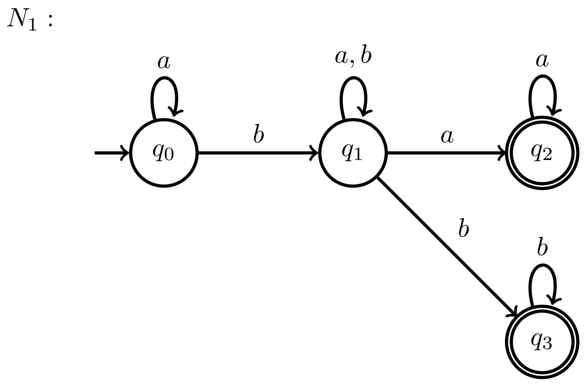
Question 4 (3 + 3 = 6 points)
Consider languages over the alphabet $\Sigma = \{0, 1\}$ with fixed length of 3.

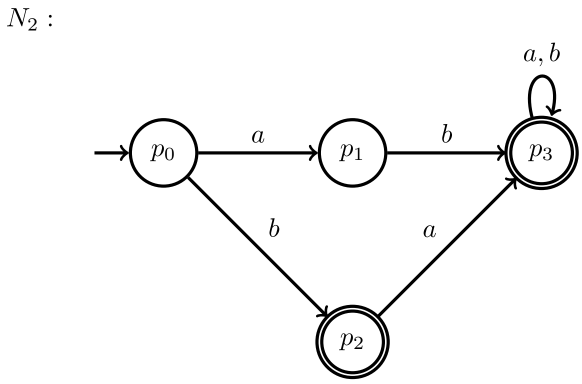
1. Let $L \subseteq \Sigma^3$ be the set of all prime numbers in the range $[0, 7]$, i.e. $\{2, 3, 5, 7\}$, represented in binary using the msbf encoding. Construct the corresponding fragment of the master automaton for L.
2. Decide whether there exists a language $L \subseteq \Sigma^3$ such that the minimal DFA for this language has at least 11 states. Explain your answer.

Question 5 $(3+2+2+4+1 = 12$ points$)$

A Büchi automaton is *very-weak* if the only cycles in the automaton (when the transition relation is viewed as a directed graph) are self-loops.

Consider now the following two very-weak NBAs N_1 and N_2:





1. Show that very-weak NBAs are closed under union and intersection by giving constructions for union and intersection. Argue that these constructions yield very-weak NBAs and the correct languages.
2. Apply the intersection construction you defined in (1) to N_1 and N_2.
3. Construct a very-weak NBA for the LTL formula $\varphi = (a\ \mathbf{U}\ \mathbf{X}b) \vee \mathbf{F}c$.
4. By LTL($\mathbf{F}$,$\mathbf{X}$) we denote the fragment of LTL with the restricted syntax of:

$$\varphi \coloneqq a \mid \neg a \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \mathbf{F}\varphi \mid \mathbf{X}\varphi$$

   Sketch a (recursive) procedure that translates LTL($\mathbf{F}$,$\mathbf{X}$) to very-weak NBAs.
5. Give an LTL formula that cannot be recognised by very-weak NBAs.

Question 6 $(2+2 = 4$ points$)$

Let $L \subseteq \Sigma^*$ be a language of finite words. We define the *limit ω-language* $\overrightarrow{L}$ of a given language L as follows:

$$w \in \overrightarrow{L} \iff \text{infinitely many prefixes of } w \text{ are in } L$$

For example, if $L = b + (ab)^*$ then $\overrightarrow{L} = (ab)^\omega$.

1. Give an NFA N such that the limit of its language and the language of N viewed as a Büchi automaton differ.
2. Prove that for any DFA D, the limit of its language and the language of D viewed as a Büchi automaton coincide.