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Automata and Formal Languages – Endterm

***Please note*: If not stated otherwise, all answers have to be justified.**

Exercise 1 **6 × 1.5 P = 9 P**

Question: Consider $L \subseteq \{a, b\}^*$ the set of words, where a occurs only at even, or only at odd positions (not necessarily at *all* even/odd positions). For example, $aba \in L, babbb \in L, aa \notin L$. Give a corresponding MSO formula *and* an automaton for L. You may use macros from the lecture notes.

Answer:

Question: Let $L \subseteq \Sigma^*$ be a language defined by the MSO sentence $\forall X \exists x\ (x \in X \vee x \notin X)$. Write down a regular expression for L.

Answer:

Question: Write an MSO formula for the language $(aab)^*$. You may use macros from the lecture notes.

Answer:

Question: Give a Büchi automaton *and* a ω-regular expression for the formula $\mathbf{F}\,\mathbf{G}\,(p \vee q)$ over the atomic propositions $AP = \{p, q\}$. (Recall that the language of the formula is an ω-language over the alphabet $\Sigma = 2^{AP}$.)

Answer:

Question: Decide whether $\mathbf{G}(a\mathbf{U}b)$ and $\mathbf{G}(a \vee b) \wedge \mathbf{F}b$ are equivalent and prove your answer.

Answer:

Question: Let $\mathcal{A}$ be a DFA recognizing a language $L \subseteq \Sigma^n$ of a fixed length n. What is the language recognized by $\mathcal{A}$ seen as a co-Büchi automaton? (Recall that transition function of a DFA is total.)

Answer:

Exercise 2

4 P

(a) Give a transducer $\mathcal{T}$ over $\{0,1\}$ recognizing the lsbf encodings of the pairs $(v,w) \in \mathbb{N} \times \mathbb{N}$ such that $v < w$, i.e. $L = \{(v,w) \mid \text{lsbf}^{-1}(v) < \text{lsbf}^{-1}(w)\}$

(b) Prove that $\mathcal{T}$ recognizes L by induction on the length of the word.

Exercise 3

6 P

Recall that $\{a^m b^n \mid m = n\}$ is not regular. Decide whether the following languages are regular or not. If yes, give a corresponding automaton or a regular expression. If no, either show it has infinitely many residuals, or use closure properties as discussed in the exercises.

(a) $L_1 = \{w \in \{a,b\}^* \mid w$ contains as many a' as b's$\}$

(b) $L_2 = \{w \in \{a,b\}^* \mid w$ contains as many ab' as ba's$\}$
(For example, in $w = abaab$ there are two ab's and one ba, hence $w \notin L_2$.)

(c) $L_3 = \{a^m b^n \mid m \leq n,\ m < 1000\}$

Exercise 4

3 P

Given a finite automaton $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ recognizing a language $L \subseteq \Sigma^*$, construct a transducer $\mathcal{T} = (Q', \Sigma', \delta', q_0', F')$ recognizing $\{(a_1 \cdots a_n, b_1 \cdots b_n) \in \Sigma^* \times \Sigma^* \mid a_1 a_2 \cdots a_n \in L \text{ and } a_1 b_1 a_2 b_2 \cdots a_n b_n \in L\}$.

Exercise 5

4 P

Consider the following program P with a binary variable x initialised to 0:

```
loop
  non-deterministically choose
      either x ← 1
      or     x ← 0
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(a) Construct a network of automata for P and x and their asynchronous product.

(b) Using the standard algorithm from the lecture decide whether $\mathbf{F}\ x = 1$ holds for P.

Exercise 6

4 P

Let $\text{Inf}(w)$ denote the set of letters that occur infinitely often in the word w. Consider the language $L = \{w \in \{a,b,c\}^\omega \mid a \in \text{Inf}(w) \Rightarrow b \notin \text{Inf}(w)\}$.

(a) Construct a *deterministic* Muller automaton for L with only two states.

(b) Construct an equivalent Rabin automaton.

Exercise 7

4 P

Given a language $L \subseteq \Sigma^*$ of finite words, we define the *limit ω-language* $\overrightarrow{L} \subseteq \Sigma^\omega$ as follows: $w \in \overrightarrow{L}$ iff infinitely many prefixes of w belong to L. For example, if $L = b + (ab)^*$ then $\overrightarrow{L} = (ab)^\omega$.

(a) Give an NFA $\mathcal{N}$ such that the limit of its language and the language of $\mathcal{N}$ viewed as a Büchi automaton differ.

(b)* Prove that for a DFA $\mathcal{D}$, the limit of its language and the language of $\mathcal{D}$ viewed as a Büchi automaton coincide.