Technische Universität München
Prof. J. Esparza / J. Křetínský
Winter term 2011/12

Name:

Matrikelnummer:

Automata and Formal Languages – Endterm

***Please note*: If not stated otherwise, all answers have to be justified.**

Exercise 1 **10×1.5 P = 15 P**

Question: Decide whether the following holds and write down why.
If L^* is a regular language, then L is regular.

Answer: ______________________________

Question: Decide whether the following holds and write down why.
If R is a non-empty regular language and $L \cap R^\omega$ is ω-regular, then L is ω-regular.

Answer: ______________________________

Question: Draw a finite automaton for $\{w \in \{a, b, c\}^* \mid$ between every a and a later b there is $c\}$.

Answer: ______________________________

Question: Let L be a regular language and r the number of its residuals. Furter, let s be the minimal possible number of states of a *non-determinstic* automaton recognizing L. What is the relationship between r and s?

Answer: ______________________________

Question : Construct a transducer recognizing the relation $\{(wa, aw) \mid w \in \{a, b\}^*\}$.

Answer :

Question : Give a regular expression for the language $L \subseteq \{a, b\}^*$ of words, in which the number of ab's is the same as the number of ba's. (Examples: $aaba \in L,\ abb \notin L$.)

Answer :

Question : Write down an ω-regular expression equivalent to $\mathbf{G}(a \vee \mathbf{F}b)$ over $Ap = \{a, b\}$.

Answer :

Question : Let $\Sigma = \{a, b\}$ be an alphabet. Is there a language $L \subseteq \Sigma^\omega$ with exactly two words such that $\Sigma^\omega \setminus L$ is ω-regular?

Answer :

Question : Construct a DFA recognizing the solution space of $2x \leq y$ in the lsbf encoding.

Answer :

Question : Decide whether the formulae $\mathbf{G}(a\mathbf{U}b)$ and $\mathbf{G}(a \vee b)$ are equivalent. Why?

Answer :

Exercise 2 **2 P**

Let $L \subseteq \Sigma^*$ and $M \subseteq \Gamma^*$ be languages. We define the *shuffle* of L and M as

$$L||M := \{u_1v_1 \cdots u_nv_n \mid n \in \mathbb{N}, u_1 \cdots u_n \in L, v_1 \cdots v_n \in M, \forall 1 \leq i \leq n\ u_i \in \Sigma^*, v_i \in \Gamma^*\}$$

Example: Let $\Sigma = \{a, b\}$, $L = \{ab\}$ and $\Gamma = \{0, 1\}$, $M = \{0, 1\}$. Then $L||M = \{ab0, a0b, 0ab, ab1, a1b, 1ab\}$.

Prove that the shuffle operation preserves regularity, i.e. if L and M are regular then so is $L||M$.

Exercise 3 **2 P**

Give regular expressions for the *complements* of the following languages over $\Sigma = \{a, b\}$ and explain your answers:

(a) $(aa + bb)^*$

(b) $(a + b)^*(aa + bb)(a + b)^*$

Exercise 4 **3 P**

(a) Let $\Sigma = \{a, b\}$. Write down all residuals of $(aa^* + \varepsilon)b\Sigma^*$.

(b) Let $\Sigma = \{a\}$. Prove that $\{a^{2^n} \mid n \in \mathbb{N}\}$ has infinitely many residuals.

Exercise 5 **4 P**

Consider the following program P with a binary variable x initialised to 0:

P:
loop
1: $x \leftarrow 1$
2: $x \leftarrow 0$

(a) Construct a newtwork of automata for P and x and their asynchronous product.

(b) Construct a Büchi automaton for the *negation* of the property $\mathbf{G}\,\mathbf{F}\, x = 0$.

(c) Using the product construction, prove that the previous property holds for P.

Exercise 6 **2 P**

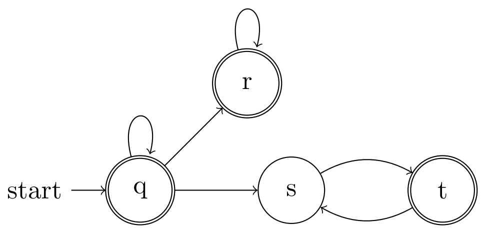
Order the following languages of infinite words over $\Sigma = \{a, b, c, d\}$ with respect to the set inclusion $\subseteq$.

- $L_1 = \{w \in \Sigma^\omega \mid \inf(w) = \{b, c\}\}$
- $L_2 = \{w \in \Sigma^\omega \mid \{b, c\} \subseteq \inf(w)\}$
- $L_3 = \{w \in \Sigma^\omega \mid \inf(w) \subseteq \{a, b, c, d\}\}$
- $L_4 = \{w \in \Sigma^\omega \mid d \notin \inf(w) \wedge \{b, c\} \subseteq \inf(w)\}$

Further, construct an *NGA* recognizing one of those (and *indicate* which one).

Exercise 7 **2 P**

Recall that *NestedDFS* and *TwoStack* are non-deterministic algorithms that stop after reporting the first accepting lasso found. Consider the following automaton:



(a) Which lassos can be reported by the *NestedDFS* algorithm? Why?

(b) Which lassos can be reported by the *TwoStack* algorithm? Why

Exercise 8 **2 P**

Consider the language L of the regular expression $(ab)^*$.

(a) Give an MSO formula for L.

(b) Give an MSO formula for L with no second-order variables.