Technische Universität München **Name:**
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Winter term 2010/11 **Matrikelnummer:**

Automata and Formal Languages – Endterm

***Please note*: If not stated otherwise, all answers have to be justified.**

Exercise 1 **each 2P=12P**

Each of the following questions admits an answer fitting in one or two lines.

(a) Prove or disprove: "Every regular language is recognized by an NFA with all states final."

(b) For every $n \in \mathbb{N}$, let us define a relation $R_n = \{(u, v) \mid lsbf^{-1}(u) = n \cdot lsbf^{-1}(v)\}$.

Assuming R_2 and R_3 are regular, prove that R_6 is regular.

(c) Let $\Sigma = \{a, b\}$ be an alphabet. Give an MSO formula defining the language of $\Sigma(a\Sigma\Sigma)^*a$.

You may use any macros defined in the "Skript".

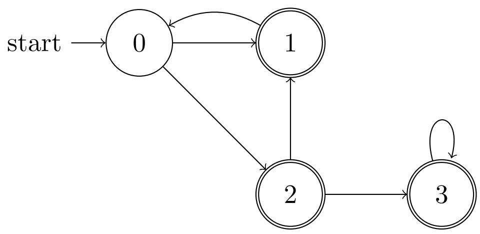
(d) Construct a Büchi automaton recognizing the following language:

$$L = \{w \in \{a, b, c, d\}^\omega \mid a, b \in \inf(w) \text{ and } d \notin \inf(w)\}$$

where $\inf(w)$ denotes the set of letters that appear infinitely many times in w.

(e) Describe a procedure to complement *deterministic* Muller automata directly without using any translation to Büchi automata.

(f) Consider the following NBA $\mathcal{A}$.



The following sequences are accepting lassos of $\mathcal{A}$. The cycles are underlined.

i) <u>010</u>

ii) <u>0210</u>

iii) 02<u>101</u>

iv) 02<u>33</u>

Which of the lassos can be found by a run of NestedDFS on $\mathcal{A}$?

Exercise 2 **4P**

Construct an eagerDFA (not a lazyDFA) for the word *mammamia* over the alphabet $\{m, a, i\}$.

Exercise 3 **5P**

Consider a variable x with domain $\{0, 1, 2\}$ initialized to 0 and the following program with two parallel processes:

Process 1:	Process 2:
loop	**loop**
1: $x \leftarrow 1$	1: $x \leftarrow 2$
2: $x \leftarrow 0$	

(a) Construct the corresponding network of the three automata and their asynchronous product.

(b) Consider a set of atomic propositions $AP = \{x = 0, x = 1, x = 2\}$. Construct a Büchi automaton over AP corresponding to the property that from some point on $x = 0$ holds forever. Give the corresponding LTL formula, too.

(c) Is there an ω-execution of the program that satisfies the property in (b)? Why?/Why not?

Exercise 4 **5P**

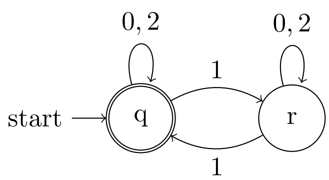
Consider languages over $\{0, 1\}$ with fixed length of 3.

(a) Construct a fragment of the master automaton for the language $L \subseteq \{0, 1\}^3$ of msbf binary encodings of all prime numbers in the range from 0 to 7. (Recall that the smallest prime number is 2.)

(b) Is there a language $L \subseteq \{0, 1\}^3$ with a minimal DFA having 10 states?

Exercise 5 **4P**

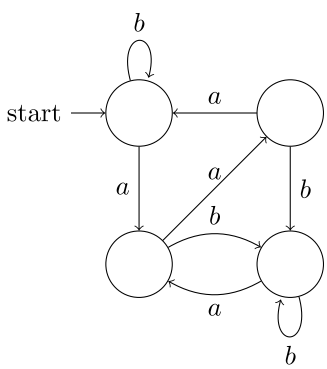
Prove that the following finite automaton over $\{0, 1, 2\}$ accepts precisely the msbf ternary encodings of even numbers. (E.g. 211 is accepted because $2 \cdot 3^2 + 1 \cdot 3^1 + 1 \cdot 3^0 = 22$ is even.) Proceed by induction on the length of the word.



Exercise 6 **4P**

A DFA is *synchronizing* if there is a word w and a state q such that after reading w from *any* state we are always in state q.

(a) Give such a word w showing that the following DFA is synchronizing.



(b) Give an algorithm to decide if a given DFA is synchronizing.

(c) Give a *polynomial time* algorithm to decide if a given DFA is synchronizing.