Technische Universität München **Winter term 2009/10**
Theoretische Informatik
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Automata and Formal Languages – Endterm

***Please note*: If not stated otherwise, all answers have to be justified.**

Exercise 1

No justifications are needed in this exercise. MSO formulae are to be interpreted over finite words for this exercise.

(a) Let $R, L \subseteq \Sigma^*$ be languages. Give a counterexample for:

If R and RL are regular, then L is regular, too.

(b) We may interpret any finite automaton $\mathcal{A}$ also as a Büchi automaton (and vice versa). Given a finite automaton $\mathcal{A}$, let $\mathcal{L}(\mathcal{A})$ denote the language of finite words represented by $\mathcal{A}$ and let $\mathcal{L}_\omega(\mathcal{A})$ denote the language of infinite words represented by $\mathcal{A}$.

- Find an automaton $\mathcal{A}$ such that $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}(\mathcal{A})^\omega$.
- Find an automaton $\mathcal{A}$ such that $\mathcal{L}_\omega(\mathcal{A}) \neq \mathcal{L}(\mathcal{A})^\omega$.

(c) Give an MSO formula φ such that $\mathcal{L}(\varphi) = \mathcal{L}((aa)^*)$ over the alphabet $\{a, b\}$. You may use the macros defined in the script.

(d) Consider the MSO formula $\varphi = \forall x\big(Q_a(x) \Rightarrow \exists y(x < y \wedge Q_b(y))\big)$. Give a regular expression r such that $\mathcal{L}(r) = \mathcal{L}(\varphi)$ over alphabet $\{a, b, c\}$.

Exercise 2

Let L be any regular language over a finite alphabet Σ.

(a) Decide (and prove) whether the following language is regular:

$$L_1 := \{x \in L \mid x \text{ starts and ends with the same letter}\}$$

(b) Show that there is a finite automaton $\mathcal{T}$ over the alphabet $\Sigma \times \Sigma$ such that

$$(x, y) \in \mathcal{L}(\mathcal{T}) \text{ iff } xy \in L \wedge |x| = |y|\,.$$

Remark: It suffices to concisely describe the idea underlying the construction of $\mathcal{T}$ in natural language.

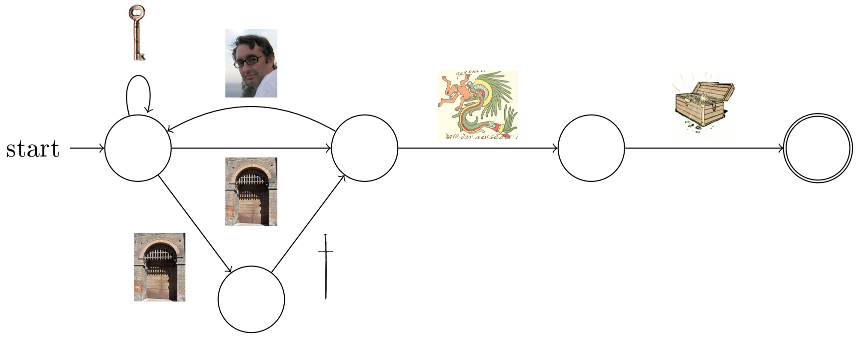
(c) Decide (and prove) whether the following language is regular:

$$L_2 := \{x \in \Sigma^* \mid \exists y \in \Sigma^* : xy \in L \wedge |x| = |y|\}$$

Remark: You may use the result of (b) independently of whether you have proven it or not.

Exercise 3

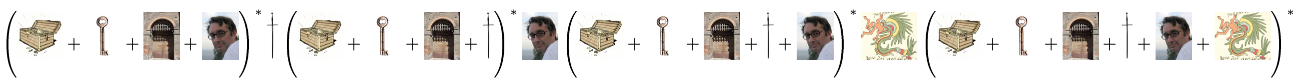
You have found a map of a long forgotten dungeon where a treasure is hidden:



You decipher the symbols on the map as follows:

 = key,  = treasure,  = gate,  = sword master,  = sword,  = dragon

(a) On the map, you find the following note written in a strange language:



- Describe in natural language the meaning of above note.

Hint: Only the relation between ,  and  really matters.

(b) Draw a finite automaton representing all paths through **any** dungeon satisfying the following requirement:

To pass through a gate, you need a key. The key is consumed when passing through a gate. At any time, you can only carry at most one key with you, i.e., any further key passed cannot be picked up.

(c) Describe in words how you can use the finite automata and constructions on them to decide whether there is a path through the dungeon which satisfies both the properties described in (a) and (b).

Exercise 4

Decide the truth value of the following Presburger formula using finite automata as discussed in the lecture:

$$\forall y\, x < y$$

Clearly state every single step of the decision procedure!

Remark: You have to stick to the algorithms discussed in the lecture. No shortcuts!

Exercise 5

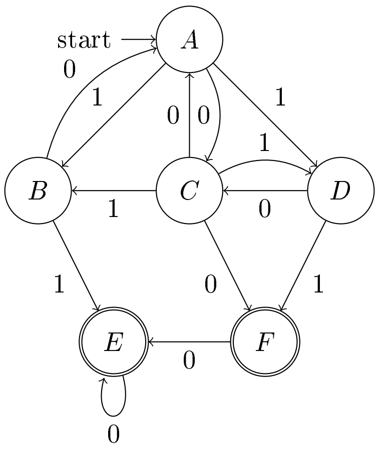
Construct a Büchi automaton recognizing the following language L over $\Sigma = \{a, b, c\}$:

$$L := \{\alpha \in \Sigma^\omega \mid \{a, b\} \subseteq \inf(\alpha) \wedge c \notin \inf(\alpha)\}.$$

Remark: It suffices to give the final automaton.

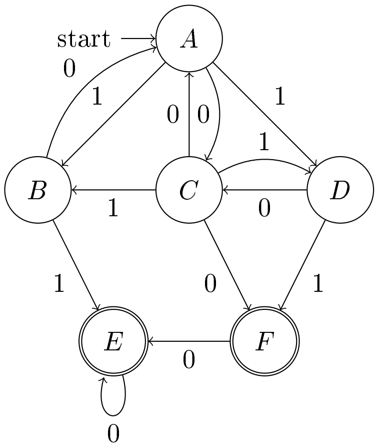
Exercise 6

Apply *Bisim* to the following automaton:

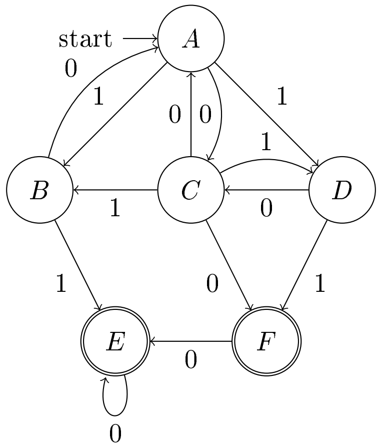


For every step of *Bisim* mark the equivalence classes in the following copies of the automaton.

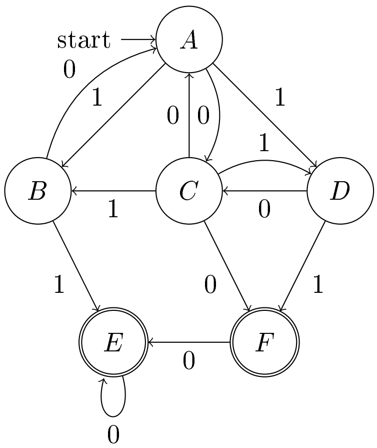
1:



2:



3:



4: