Exercise 13.1.

Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. Give LTL formulas for the following $\omega$-languages:

(a) $\{p, q\} \emptyset \Sigma^\omega$
(b) $\Sigma^* \{q\}^\omega$
(c) $\Sigma^*(\{p\} + \{p, q\}) \Sigma^* \{q\} \Sigma^\omega$
(d) $\{p\}^* \{q\}^* \emptyset^\omega$

In (a) and (d) the $\emptyset$ symbol stands for the letter $\emptyset \in \Sigma$, and not for the empty $\omega$-language.

Exercise 13.2.

Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. Give Büchi automata for the $\omega$-languages over $\Sigma$ defined by the following LTL formulas:

(a) $XG\neg p$
(b) $(GFp) \rightarrow (Fq)$
(c) $p \land \neg (XFp)$
(d) $G(p \lor (p \rightarrow q))$
(e) $Fq \rightarrow (\neg q \lor (\neg q \land p))$

Exercise 13.3.

Say which of the following equivalences hold. For every equivalence that does not hold give an instantiation of $\varphi$ and $\psi$ together with a computation that disproves the equivalence.

(a) $G\varphi \rightarrow F\varphi$
(b) $(G\varphi \land \psi) \rightarrow (G\varphi \land F\psi)$
(c) $G(\varphi \lor \psi) \equiv G\varphi \lor G\psi$
(d) $(\varphi \land \psi) \lor (\psi \lor \rho) \equiv (X(\varphi \lor \psi) \lor (\psi \lor \rho))$

Exercise 13.4.

Let $AP = \{p, q\}$ and let $\Sigma = 2^{AP}$. An LTL formula is a tautology if it is satisfied by all computations. Which of the following LTL formulas are tautologies?

(a) $Gp \rightarrow Fp$
(b) $G(p \rightarrow q) \rightarrow (Gp \rightarrow Gq)$
(c) $FGp \lor FG\neg p$
(d) $\neg Fp \rightarrow F\neg Fp$