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Automata and Formal Languages

Winter Term 2023/24 – Exercise Sheet 13

Exercise 13.1.

Let $\mathrm{AP} = \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. Give LTL formulas for the following ω-languages:

(a) $\{p, q\}\,\emptyset\,\Sigma^\omega$

(b) $\Sigma^*\,\{q\}^\omega$

(c) $\Sigma^*\,(\{p\} + \{p, q\})\,\Sigma^*\,\{q\}\,\Sigma^\omega$

(d) $\{p\}^*\,\{q\}^*\,\emptyset^\omega$

In (a) and (d) the $\emptyset$ symbol stands for the letter $\emptyset \in \Sigma$, and not for the empty ω-language.

Exercise 13.2.

Let $\mathrm{AP} = \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. Give Büchi automata for the ω-languages over Σ defined by the following LTL formulas:

(a) $\mathbf{XG}\neg p$

(b) $(\mathbf{GF}p) \to (\mathbf{F}q)$

(c) $p \wedge \neg(\mathbf{XF}p)$

(d) $\mathbf{G}(p\ \mathbf{U}\ (p \to q))$

(e) $\mathbf{F}q \to (\neg q\ \mathbf{U}\ (\neg q \wedge p))$

Exercise 13.3.

Say which of the following equivalences hold. For every equivalence that does not hold give an instantiation of φ and ψ together with a computation that disproves the equivalence.

(a) $\mathbf{F}(\varphi \vee \psi) \equiv \mathbf{F}\varphi \vee \mathbf{F}\psi$

(b) $\mathbf{F}(\varphi \wedge \psi) \equiv \mathbf{F}\varphi \wedge \mathbf{F}\psi$

(c) $\mathbf{G}(\varphi \vee \psi) \equiv \mathbf{G}\varphi \vee \mathbf{G}\psi$

(d) $(\varphi \vee \psi)\ \mathbf{U}\ \rho \equiv (\varphi\ \mathbf{U}\ \rho) \vee (\psi\ \mathbf{U}\ \rho)$

(e) $\mathbf{GF}(\varphi \wedge \psi) \equiv \mathbf{GF}\varphi \wedge \mathbf{GF}\psi$

(f) $\mathbf{X}(\varphi\ \mathbf{U}\ \psi) \equiv (\mathbf{X}\varphi\ \mathbf{U}\ \mathbf{X}\psi)$

Exercise 13.4.

Let $\mathrm{AP} = \{p, q\}$ and let $\Sigma = 2^{\mathrm{AP}}$. An LTL formula is a tautology if it is satisfied by all computations. Which of the following LTL formulas are tautologies?

(a) $\mathbf{G}p \to \mathbf{F}p$

(b) $\mathbf{G}(p \to q) \to (\mathbf{G}p \to \mathbf{G}q)$

(c) $\mathbf{FG}p \vee \mathbf{FG}\neg p$

(d) $\neg\mathbf{F}p \to \mathbf{F}\neg\mathbf{F}p$

(e) $(\mathbf{G}p \to \mathbf{F}q) \leftrightarrow (p\ \mathbf{U}\ (\neg p \vee q))$

(f) $\neg(p\ \mathbf{U}\ q) \leftrightarrow (\neg p\ \mathbf{U}\ \neg q)$

(g) $\mathbf{G}(p \to \mathbf{X}p) \to (p \to \mathbf{G}p)$