Exercise 11.1.

Give generalized Büchi automata (NGA) for the following ω-languages:

- \( L_1 = \{ w \in \{a, b, c\}^\omega : w \text{ contains infinitely many } a\text{'s and } b\text{'s and } c\text{'s.} \}, \)
- \( L_2 = \{ w \in \{a, b, c\}^\omega : w \text{ contains finitely many } b\text{'s} \}, \)

Intersect these automata and decide if the obtained automaton is the smallest generalized Büchi automaton for \( L_1 \cap L_2 \) in terms of number of states.

Exercise 11.2.

Give an algorithm that directly complements deterministic Muller automata, without going through Büchi automata. Is this algorithm “practical”?

Exercise 11.3.

(a) Consider the following Büchi automaton \( A \) over \( \Sigma = \{a, b\} \):

\[
\begin{array}{c}
\text{a, b} \\
\rightarrow \text{q0} \\
\text{b} \\
\rightarrow \text{q1} \\
\end{array}
\]

\( a, b \)

Draw \( \text{dag}(abab^\omega) \) and \( \text{dag}((ab)^\omega) \).

(b) Let \( r_w \) be the ranking of \( \text{dag}(w) \) defined by

\[
r_w(q, i) = \begin{cases} 
1 & \text{if } q = q_0 \text{ and } (q_0, i) \text{ appears in } \text{dag}(w), \\
0 & \text{if } q = q_1 \text{ and } (q_1, i) \text{ appears in } \text{dag}(w), \\
\perp & \text{otherwise}. 
\end{cases}
\]

Are \( r_{abab^\omega} \) and \( r_{(ab)^\omega} \) (over \( A \)) odd rankings?

(c) Consider the following Büchi automaton \( B \) over \( \Sigma = \{a, b\} \):

\[
\begin{array}{c}
\text{a} \\
\rightarrow \text{q0} \\
\text{a} \\
\rightarrow \text{q1} \\
\text{a} \\
\rightarrow \text{q2} \\
\end{array}
\]

\( a \)

Draw \( \text{dag}(a^\omega) \). Show that any odd ranking for this dag must contain a node of rank 3 or more.

(d) Consider again the automaton \( A \) from (a). Let \( w \) be an ω-word and \( r_w \) the ranking of \( \text{dag}(w) \) as defined in (b). Show that \( r_w \) is an odd ranking for \( \text{dag}(w) \) if and only if \( w \notin L_\omega(A) \).

(e) Construct a Büchi automaton accepting \( L_\omega(A) \) using the construction seen in class. \textit{Hint:} by (d), it is sufficient to use \( \{0, 1\} \) as ranks.
Exercise 11.4.
Show that for every DBA $A$ with $n$ states there is an NBA $B$ with $2n$ states such that $B = \overline{A}$. Explain why your construction does not work for NBAs.