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Automata and Formal Languages

Winter Term 2023/24 – Exercise Sheet 11

Exercise 11.1.

Give *generalized* Büchi automata (NGA) for the following ω-languages:

- $L_1 = \{w \in \{a, b, c\}^\omega : w$ contains infinitely many a's and b's and c's.$\}$,
- $L_2 = \{w \in \{a, b, c\}^\omega : w$ contains finitely many b's$\}$,

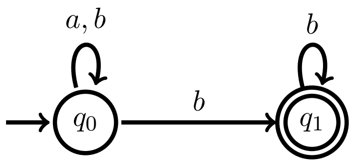
Intersect these automata and decide if the obtained automaton is the smallest generalized Büchi automaton for $L_1 \cap L_2$ in terms of number of states.

Exercise 11.2.

Give an algorithm that directly complements deterministic Muller automata, without going through Büchi automata. Is this algorithm "practical"?

Exercise 11.3.

(a) Consider the following Büchi automaton A over $\Sigma = \{a, b\}$:



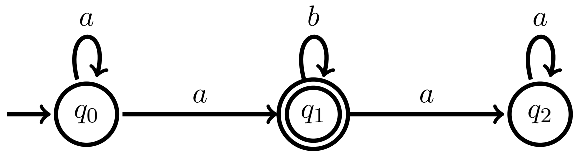
Draw dag($abab^\omega$) and dag($(ab)^\omega$).

(b) Let r_w be the ranking of dag(w) defined by

$$r_w(q, i) = \begin{cases} 1 & \text{if } q = q_0 \text{ and } \langle q_0, i \rangle \text{ appears in dag}(w), \\ 0 & \text{if } q = q_1 \text{ and } \langle q_1, i \rangle \text{ appears in dag}(w), \\ \bot & \text{otherwise.} \end{cases}$$

Are r_{abab^ω} and $r_{(ab)^\omega}$ (over A) odd rankings?

(c) Consider the following Büchi automaton B over $\Sigma = \{a, b\}$:



Draw dag(a^ω). Show that any odd ranking for this dag must contain a node of rank 3 or more.

(d) Consider again the automaton A from (a). Let w be an ω-word and r_w the ranking of dag(w) as defined in (b). Show that r_w is an odd ranking for dag(w) if and only if $w \notin L_\omega(A)$.

(e) Construct a Büchi automaton accepting $\overline{L_\omega(A)}$ using the construction seen in class. *Hint*: by (d), it is sufficient to use $\{0, 1\}$ as ranks.

Exercise 11.4.

Show that for every DBA A with n states there is an NBA B with $2n$ states such that $B = \overline{A}$. Explain why your construction does not work for NBAs.