Prof. Javier Esparza
Philipp Czerner

Technical University of Munich
Chair for Foundations of Software Reliability

Automata and Formal Languages

Winter Term 2023/24 – Exercise Sheet 10

Exercise 10.1.

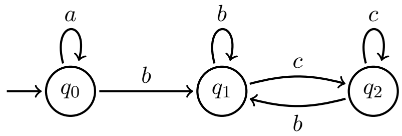
Consider automata with the set of states $Q = \{q_0, q_1, q_2\}$ and the acceptance conditions $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ given by the following table:

	$\{q_0\}$	$\{q_1\}$	$\{q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
α_1	1	0	0	1	1	0	1
α_2	0	1	0	1	0	0	0
α_3	1	1	0	1	0	0	0
α_4	0	0	0	0	0	0	1

(a) For each of the conditions determine if they are Büchi, co-Büchi, Rabin, Muller.

(b) Can it happen that an accepting condition is neither Büchi nor co-Büchi nor Rabin nor Muller? If yes, give an example of such a condition.

(c) Consider the following automaton with acceptance conditions $\alpha_1, \alpha_2, \alpha_3, \alpha_4$. What are the languages accepted by the obtained automata?



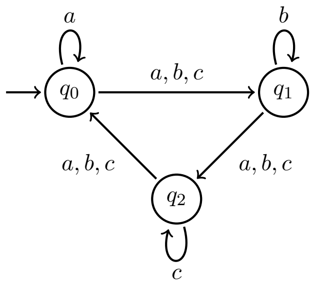
Exercise 10.2.

Let language $L = \{w \in \{a, b\}^\omega : w \text{ contains finitely many } a\}$

(a) Give a deterministic Rabin automaton for L.

(b) Give an NBA for L and try to "determinize" it by using the NFA to DFA powerset construction. What is the language accepted by the resulting DBA?

(c) What ω-language is accepted by the following Muller automaton with acceptance condition $\{\{q_0\}, \{q_1\}, \{q_2\}\}$? And with acceptance condition $\{\{q_0, q_1\}, \{q_1, q_2\}, \{q_2, q_0\}\}$?



Exercise 10.3.

Let $L_1 = (ab)^\omega$ and let L_2 be the language of all words over $\{a, b\}$ containing infinitely many a and infinitely many b.

(a) Exhibit three different DBAs with three states recognizing L_1.

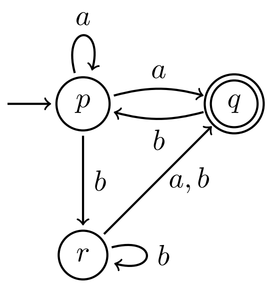
(b) Exhibit six different DBAs with three states recognizing L_2.

(c) Show that no DBA with at most two states recognizes L_1 or L_2.

Exercise 10.4.

(a) Show that for every NCA there is an equivalent NBA.

(b) For the following NCA give an equivalent NBA, using the construction from (a):



Exercise 10.5.

Give a procedure that translates non-deterministic Rabin automata to non-deterministic Büchi automata.