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Automata and Formal Languages

Winter Term 2023/24 – Exercise Sheet 3

Exercise 3.1.

Analyse the residuals of the following languages. If there are finitely many of them, determine them; otherwise prove that there are infinitely many of them.

(a) $(a + bbc)^*$ over $\Sigma = \{a, b, c\}$,

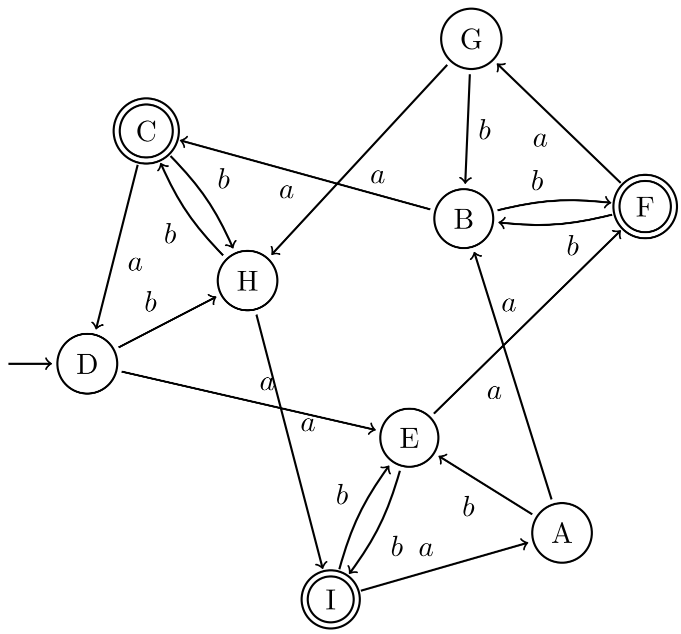
(b) $(aa)^*$ over $\Sigma = \{a, b\}$,

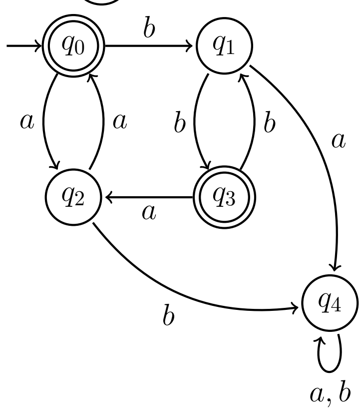
(c) $\{a^n b^{n+1} \mid n \geq 0\}$ over $\Sigma = \{a, b\}$,

(d) $\{a^{2^n} \mid n \geq 0\}$ over $\Sigma = \{a\}$.

Exercise 3.2.

Let A and B be respectively the following DFAs:





(a) Compute the language partitions of A and B.

(b) Construct the quotients of A and B with respect to their language partitions.

(c) Give regular expressions for $L(A)$ and $L(B)$.

Exercise 3.3.

Given $n \in \mathbb{N}$, let $\mathrm{MSBF}(n)$ be the set of *most-significant-bit-first* encodings of n, i.e., the words that start with an arbitrary number of leading zeros, followed by n written in binary. For example:

$$\mathrm{MSBF}(3) = 0^*11 \quad \text{and} \quad \mathrm{MSBF}(9) = 0^*1001 \quad \mathrm{MSBF}(0) = 0^*$$

Similarly, let $\mathrm{LSBF}(n)$ denote the set of *least-significant-bit-first* encodings of n, i.e., the set containing for each word $w \in \mathrm{MSBF}(n)$ its reverse. For example:

$$\mathrm{LSBF}(6) = 0110^* \quad \text{and} \quad \mathrm{LSBF}(0) = 0^*$$

For any $n \geq 2$, let $M_n = \{w \in \{0,1\}^* \mid w \in \mathrm{MSBF}(k) \text{ and } k \text{ is a multiple of } n\}$ and $L_n = \{w \in \{0,1\}^* \mid w \in \mathrm{LSBF}(k) \text{ and } k \text{ is a multiple of } n\}$.

In the following, let $p > 2$ be any prime number.

(a) Prove that M_p and L_p have at least p many residuals.

(b) Give the minimal DFA A_p (with p states) for the language M_p.

(c) Prove that the NFA obtained by reversing the transitions of A_p and swapping the initial and final states is a DFA. Conclude that the minimal DFA for L_p has p states.