Petri nets — Exercise sheet 5

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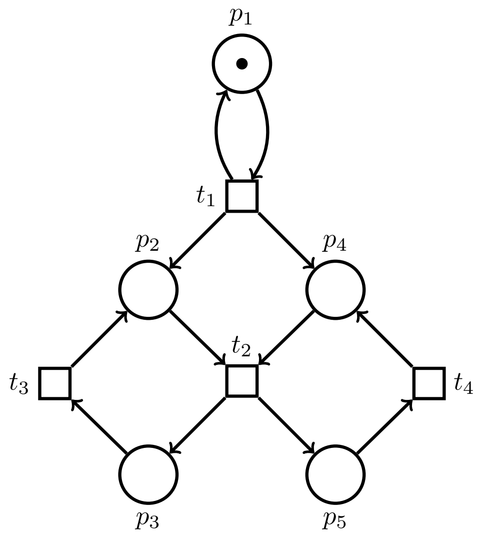
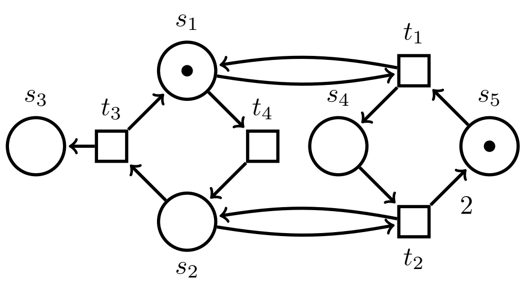
Exercise 5.1

(a) Show that
$$X = \{(x_1, x_2, x_3) \in \mathbb{N}^3 : (x_1 + 3 \le x_2 \le x_3 + 1) \vee (x_2 = 2x_1 + x_3 + 5)\}$$
is semilinear by giving its representation as a finite set of roots and periods.

(b) Describe the set of reachable markings of the following Petri net as a semi-linear set, with the markings seen as vectors of $\mathbb{N}^5$.



Exercise 5.2
We want to show that the following Petri net with weighted arcs has a non-semilinear reachability set.



Consider the following sets of markings, given as $M = (s_1, s_2, s_3, s_4, s_5)$:

$$\mathcal{M}_1 = \{(1, 0, x_1, x_2, x_3) \mid 0 < x_2 + x_3 \le 2^{x_1}\}$$
$$\mathcal{M}_2 = \{(0, 1, x_1, x_2, x_3) \mid 0 < 2x_2 + x_3 \le 2^{x_1+1}\}$$
$$\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$$

The set $\mathcal{M}$ is non-semilinear. We are going to show that $\mathcal{M}$ is equal to the set of reachable markings for the above Petri net.

1. Show that if $M_0 \xrightarrow{*} M$, then $M \in \mathcal{M}$. For this, show that $M_0 \in \mathcal{M}$ and if $M \in \mathcal{M}$ and $M \xrightarrow{t} M'$ for some transition t, then also $M' \in \mathcal{M}$.
2. ★ Show that if $M \in \mathcal{M}$, then $M_0 \xrightarrow{*} M$.
   Hint: Do this by induction on $x_1 = M(s_3)$ for $M \in \mathcal{M}$. In the induction step at x_1, do a case distinction between $M \in \mathcal{M}_1$ and $M \in \mathcal{M}_2$. In each case, find an M' for which you can apply the induction hypothesis and from which M is reachable.

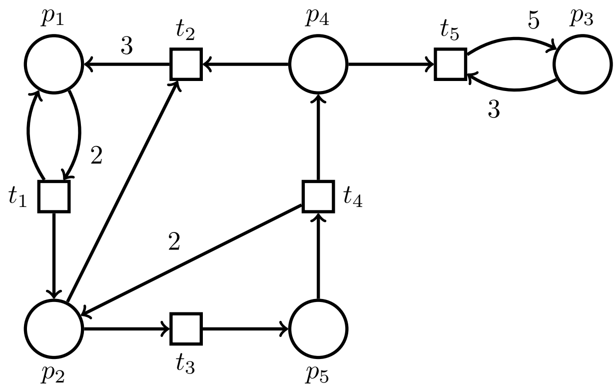
Exercise 5.3

Give a family of bounded Petri nets $\{N_k\}_{k \in \mathbb{N}}$ such that the size of N_k is bounded by $O(k)$ (that is, there is a $c \in \mathbb{N}$ such that for all $N_k = (S, T, F, M_0)$, we have $|S| + |T| + |F| \leq ck$ and $\forall s \in S : M_0(s) \leq ck$), but each N_k has a reachable marking M and a place s with $M(s) \geq 2^{2^k}$.

Hint: Construct a net that doubles the number of tokens in a place. Modify it so that one occurrence sequence for doubling removes exactly one token from a certain place. Use this construct again or the construct from the lecture to put 2^k tokens into that place.

Exercise 5.4

Consider the following Petri net (with weights) $\mathcal{N}$:



(a) Build the incidence matrix of $\mathcal{N}$.

(b) Let $M_0 = \{p_1, p_1\}$. Try to determine whether

$$M_0 \xrightarrow{*} \{p_1, p_1, p_1, p_4\},$$
$$M_0 \xrightarrow{*} \{p_1, p_1, p_1, p_1, p_2\},$$
$$M_0 \xrightarrow{*} \{p_1, p_2, p_5\},$$

by solving the marking equation.