Technische Universität München, I7 **Summer Term 2020**
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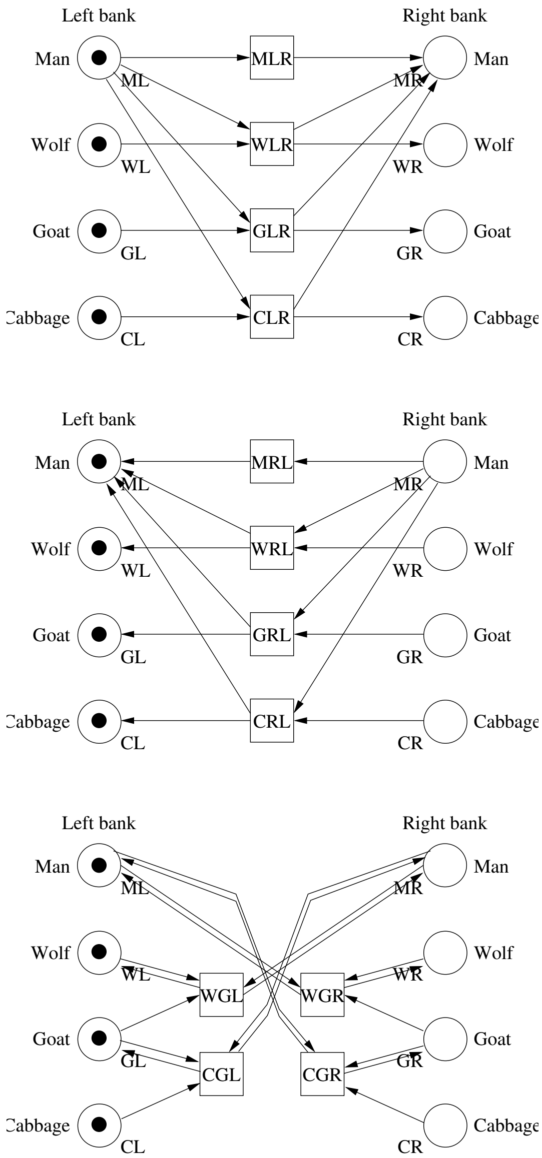
Petri nets — Exercise sheet 2

Solution to be published on 12.05.2020

Exercise 2.1

Recall the problem of the man who wants to bring a wolf, a goat and a cabbage across a river in his two-seat boat, presented in Chapter 2.4 of the lecture notes (video CH2V4).



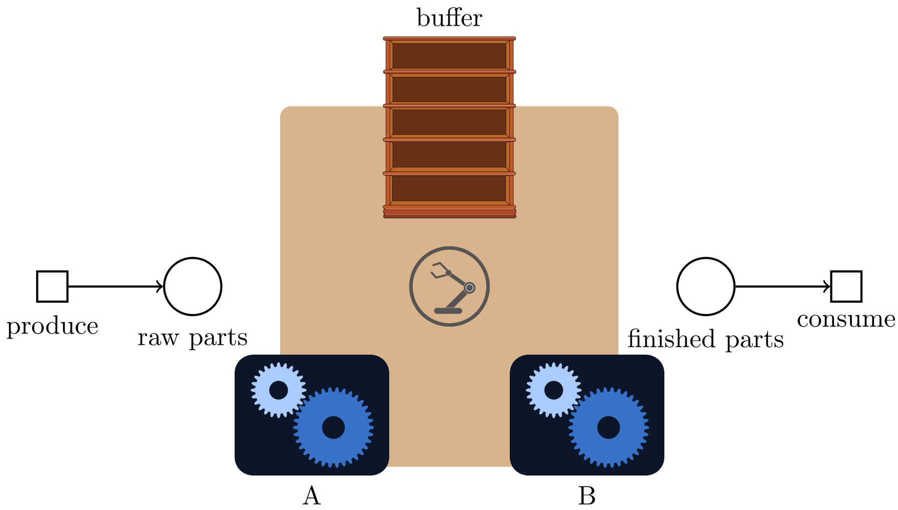
The Petri net $(\mathcal{N}, M_L)$ above models the problem, and you can play with it in `PIPE` with the file `WolfGoatCabbage.xml`. We number the places from 0 to 7, from top to bottom and then from left to right. For example, the place modelling that the man is on the left is number 0 and the place modelling that the wolf is on the right is number 5. Marking $M_L = (1,1,1,1,0,0,0,0)$ corresponds to the man, wolf, goat and cabbage being on the left side. We are interested in the paths leading from M_L to $M_R = (0,0,0,0,1,1,1,1)$.

(a) First, let us assume the wolf and goat are well-behaved: even when they are left alone (without the man), the wolf does not eat the goat and the goat does not eat the cabbage. Show that it is possible for the man to transfer wolf, goat and cabbage from the left bank to the right bank by giving a short firing sequence from M_L to M_R.

(b) Now imagine the animals are not well-trained and very hungry: left alone (without the man), the wolf *will* eat the goat and the goat *will* eat the cabbage. Draw the reachability graph of $(\mathcal{N}, M_L)$ omitting all markings in which the wolf can eat the goat, as well as the markings in which the goat can eat the cabbage. Is M_R a marking of the reachability graph? If yes, give a firing sequence leading from M_L to M_R such that the animals can never eat.

Exercise 2.2 (adapted from [2, ex. 2.20])

Consider a simple production system in which raw parts are first processed by a machine A, stored into a buffer, and then processed by a machine B. The parts are moved around using a single robot arm R. The buffer can contain at most five items at a time, and machines A and B can only handle one item at a time.

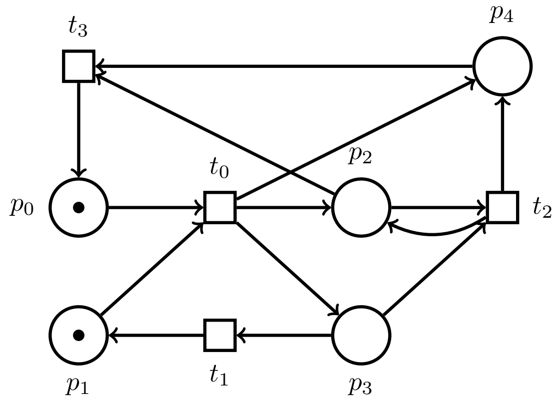


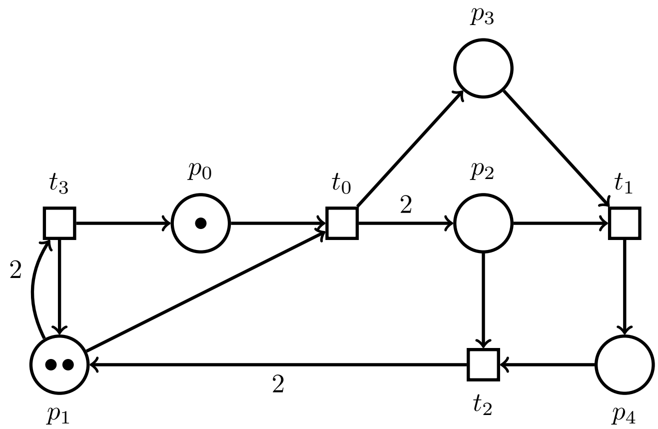
Model this production system as a Petri net by extending the partial model shown above. The actions of machines A and B are not atomic: they have a beginning and an end. On the other hand, the action of the robot arm can be considered as atomic. There is no need to distinguish between particular buffer places, or between particular items to be processed.

Exercise 2.3

(a) The first Petri net is the same as in Exercise 1.1 of the last exercise sheet. Using the reachability graph you constructed, say whether $(\mathcal{N}, M_0)$ is bounded, deadlock-free and/or live. If it is bounded, give the smallest k such that it is k-bounded. Justify your answers.

(b) Construct the reachability graph of the second Petri net. Notice that it is a weighted Petri net. Say whether it is bounded, deadlock-free and/or live. If it is bounded, give the smallest k such that it is k-bounded. Justify your answers.





Exercise 2.4
In video CH2V8, from 17:50 on, 8 different Petri nets are shown from the paper by Murata [1] such that they exhibit all possible combinations of the following three properties and their negations:

- bounded (B) / not bounded ($\overline{B}$)
- live (L) / not live ($\overline{L}$)
- reversible (R) / not reversible ($\overline{R}$)

A Petri net $(\mathcal{N}, M_0)$ is said to be reversible if, for each marking M reachable from M_0, marking M_0 is reachable from M. In other words, in an reversible net it is always possible to return to the initial marking.

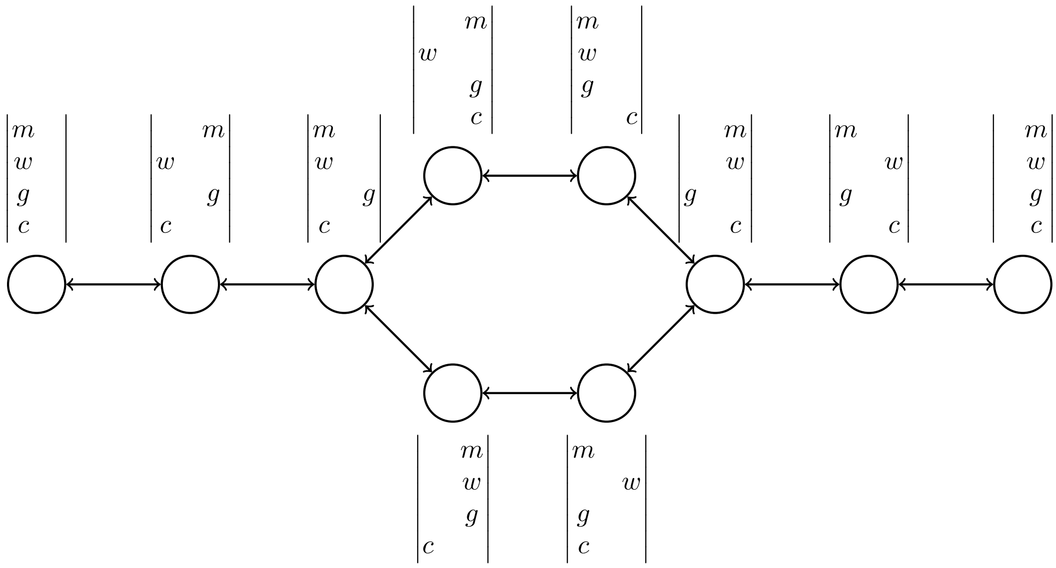
(a) Without looking at the video or the paper, for each of the 8 possible combinations, try to find a Petri net exhibiting this combination (have at least one place and at least one transition).
Hint: The bounded, live, but not reversible Petri net is hard to find. However, it can be done with only 4 places and 3 transitions.

(b) Compare your solutions to the Petri nets in the paper. Actually, none of the solutions given by Murata are minimal. Can you find smaller solutions?

Solution 2.1

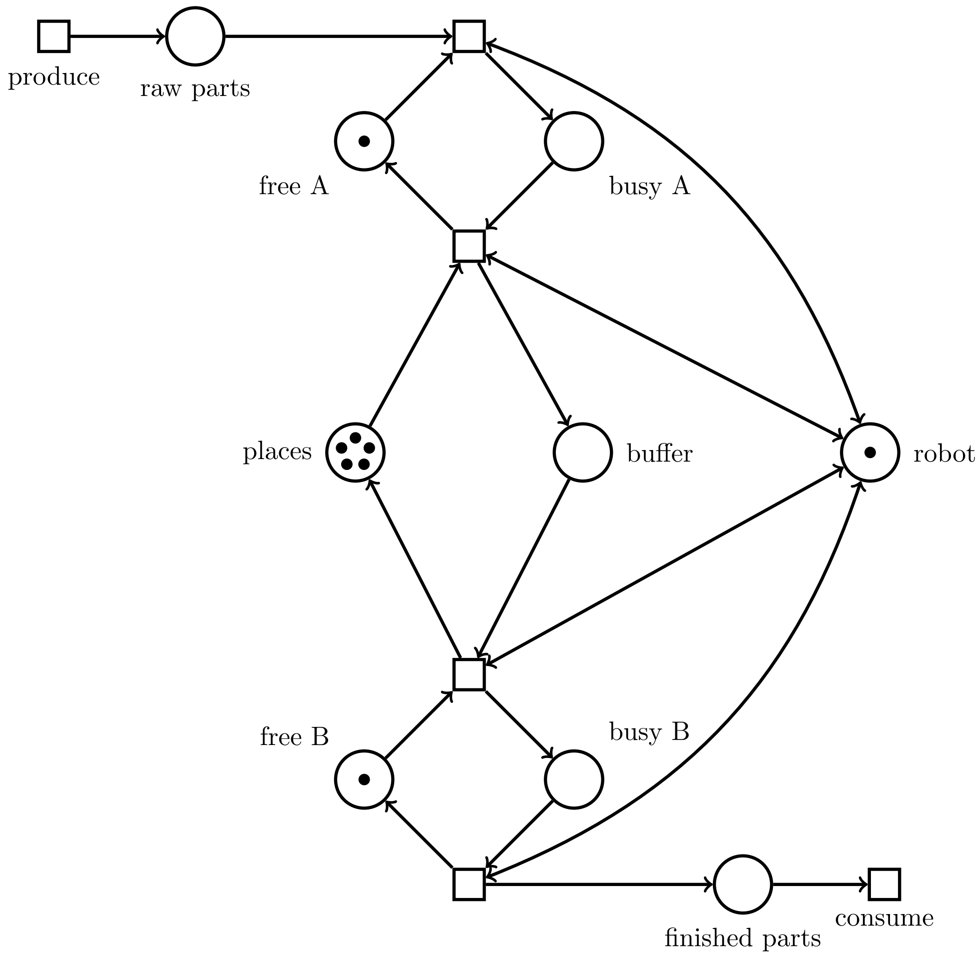
(a) A firing sequence: WLR,MRL,GLR,MRL,CLR.

(b) The reachability graph without markings in which animals can eat:



Marking M_R is reachable, and the trick is for the man to bring the goat back to the left shore with him after he has brought either the wolf or the cabbage to the right shore. Firing sequence GLR, MRL, WLR, GRL, CLR, MRL, GLR goes from M_L to M_R.

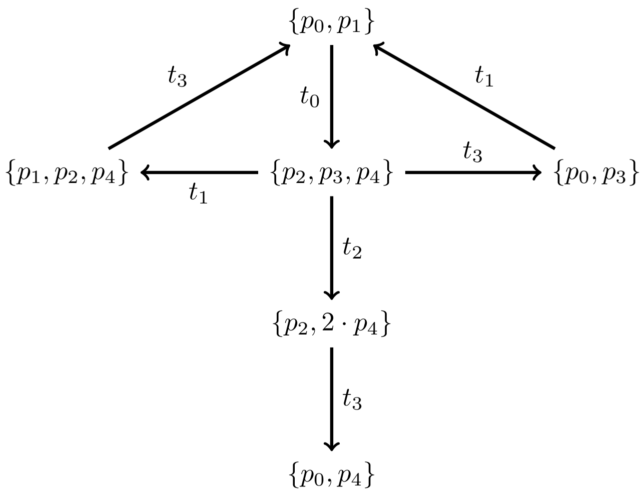
Solution 2.2 (adapted from [2, ex. 2.20])
The production system can be modelled as follows:



Notice that since the robot place is always marked, it could also be omitted.

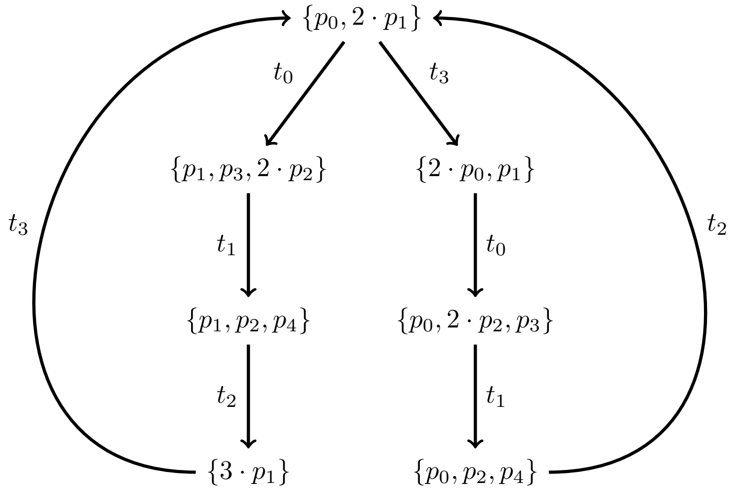
Solution 2.3

1. (a)



   (b) It is 2-*bounded* since all markings of the reachability graph have at most two tokens in each place. It is *not deadlock-free* since $\{p_0, p_4\}$ has no successor. It is *not live* since it is not deadlock-free.

2. (a)

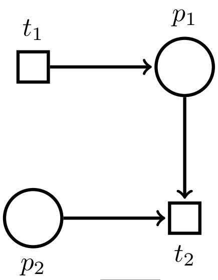


(b) It is 3-*bounded* since all markings of the reachability graph have at most three tokens in each place. It is *deadlock-free* since every marking of the reachability graph has an outgoing arc. It is *live* because for every transition t, every marking M of the reachability graph leads to a marking M' with an outgoing arc labeled by t.

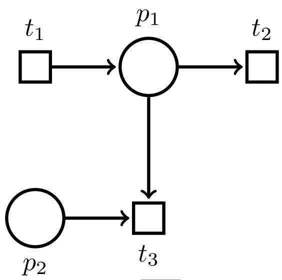
★ Alternatively, liveness follows from the fact that the reachability graph is strongly connected and has an occurrence of every transition.

Solution 2.4

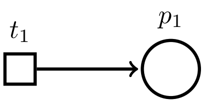
The following 8 Petri nets exhibit the desired properties. The labels (a)-(h) correspond to the same labels in the paper by Murata and show that the nets given here are smaller.



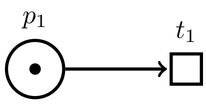
(a) $\overline{B}\overline{L}\overline{R}$



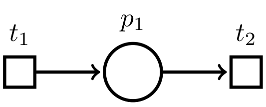
(b) $\overline{B}\overline{L}R$



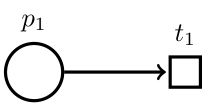
(c) $\overline{B}L\overline{R}$



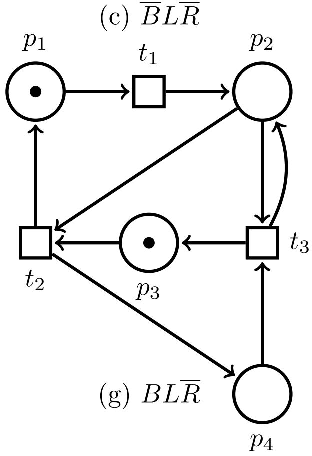
(d) $B\overline{L}\overline{R}$



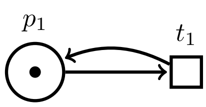
(e) $\overline{B}LR$



(f) $B\overline{L}R$



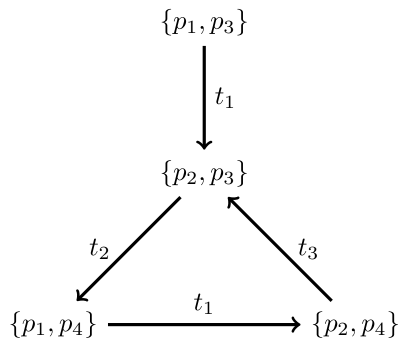
(g) $BL\overline{R}$



(h) BLR

For most Petri nets, it is clear that they have the desired properties: for (d), (f) and (h), the reachability graph just has one or two markings; while for (a), (b), (c) and (e), the reachability graph is infinite, but essentially a line with a marking for each natural number, traversable only in forward direction for (a) and (c) and in both directions for (b) and (e).

To see that the Petri net (g) for the combination $BL\overline{R}$ is indeed bounded, live, but not reversible, we can construct the reachability graph:



The reachability graph is not strongly connected: after firing t_1, we can not return to the initial marking. Therefore the Petri net is not reversible. However, the strongly connected component reached after firing t_1 contains every transition, so the Petri net is live. As the reachability graph is finite, the Petri net is bounded.

References

[1] Tadao Murata. Petri nets: Properties, analysis and applications. *Proceedings of the IEEE*, 77(4):541–580, April 1989. Available at `http://www2.ing.unipi.it/~a009435/issw/extra/murata.pdf`.

[2] Wil van der Aalst, Massimiliano de Leoni, Boudewijn van Dongen, and Christian Stahl. Course business information systems: Exercises. Available at `http://wwwis.win.tue.nl/~wvdaalst/old/courses/BIScourse/exercise-bundle-BIS-2015.pdf`, 2015.