Chair of Foundations of Software Reliability and Theoretical Computer Science
Department of Informatics
Technical University of Munich

Personal sticker

S5002

Petri nets

Exam: IN2052 / Endterm **Date:** Thursday 6th August, 2020
Examiner: Javier Esparza **Time:** 14:15 – 15:30

Working instructions

- This exam consists of **4 pages** with a total of **3 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 40 credits.
- Allowed resources:
  - any electronic resources accessible using only the external mouse
- You have **75 minutes** to complete the exam.
- All answers have to be written on your own paper.
- Only write on one side of each sheet of paper.
- Write with black or blue pen on white DIN A4 paper.
- Write your name and immatriculation number on every sheet.
- Write a consecutive page number on every sheet.
- Note that we sometimes represent a marking M by the tuple $(M(p_1), M(p_2), \ldots, M(p_n))$.
- ★ denotes a harder question.

☐ Exam empty

Problem 1 Statement (0 credits)

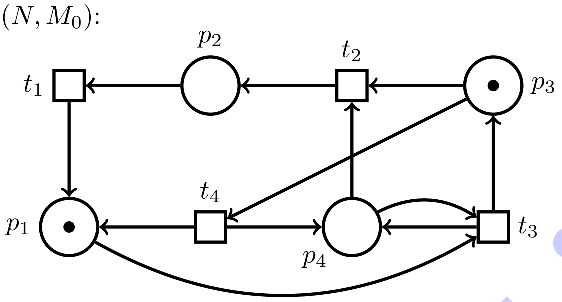
It is MANDATORY that your answer sheet includes a signed copy of the following this statement.

"I did not communicate with anyone during this graded exercise and only used the allowed resources."

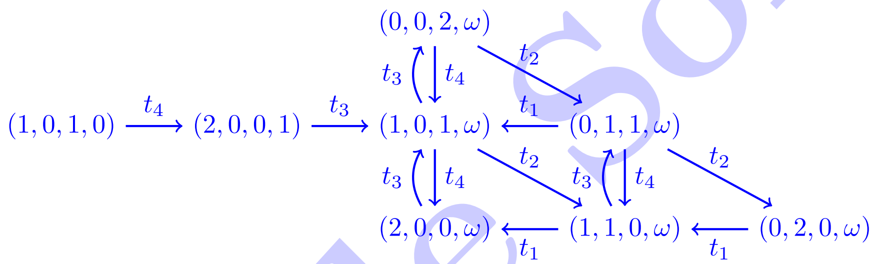
Signed: ______________________________

Problem 2 (30 credits)

Consider the following Petri net (N, M_0):



a) Draw the coverability graph of (N, M_0).
Hint: Our solution has 8 ω-markings. (6 credits)



b) Give the bound of each place in (N, M_0), where the bound is ω if the place is unbounded. (2 credits)

$p_1 : 2 \quad p_2 : 2 \quad p_3 : 2 \quad p_4 : \omega$

c) Is (N, M_0) deadlock free? If yes, give a proof showing that (N, M_0) is deadlock free; if not, give a firing sequence leading to a dead marking. (3 credits)

(N, M_0) is not deadlock free. The following sequence leads to a dead marking:

$$(1,0,1,0) \xrightarrow{t_4} (2,0,0,1) \xrightarrow{t_3} (1,0,1,1) \xrightarrow{t_2} (1,1,0,0) \xrightarrow{t_1} (2,0,0,0)$$

d) Give both:

- a semi-positive S-invariant I of N where $I(p_2) = 1$ and $I(p_3) = 1$; and
- a semi-positive T-invariant J of N where $J(t_3) = 2$.

(4 credits)

$I = (1,1,1,0)$
$J = (1,1,2,1)$

e) Give all the minimal proper traps of N.
Hint: Every minimal proper trap of N contains the place p_1. (2 credits)

$\{p_1, p_2, p_3\}, \{p_1, p_2, p_4\}$

f) Let $M = \{2 \cdot p_3\} = (0, 0, 2, 0)$. Using only S-invariants of N, can you decide whether M is reachable from M_0? Justify your answer.
Hint: You may use without proof that the vector space of S-invariants of N has dimension 1. (4 credits)

Since the vector space has dimension one, the S-invariant $I = (1, 1, 1, 0)$ forms a basis of the space of S-invariants of N. As $I \cdot M_0 = I \cdot M = 2$, we then have that $M_0 \sim M$. This could hold both for reachable and unreachable markings. Therefore we can not decide whether M is reachable from M_0 using only S-invariants.

g) Consider the same $M = \{2 \cdot p_3\} = (0, 0, 2, 0)$. Using only traps of N, can you decide whether M is reachable from M_0? Justify your answer. (4 credits)

Yes, we can decide this using the trap $R = \{p_1, p_2, p_4\}$. We have $M_0(R) = 1$ and $M(R) = 0$, so M_0 marks the trap, but M does not. If M would be reachable, this would violate the fundamental property of traps, so we can decide that M is not reachable from M_0 using traps.

h) ★ The minimal proper siphons of N are $R_1 = \{p_1, p_2, p_3\}$ and $R_2 = \{p_3, p_4\}$.
Show that for any marking M, there is a marking M' reachable from M such that R_1 or R_2 is empty at M'.
Show that there is no initial marking M such that (N, M) is live. (5 credits)

Let M be an arbitrary marking. If $M(R_1) = 0$ for the trap and siphon $R_1 = \{p_1, p_2, p_3\}$, then we are done. Now assume $M(R_1) > 0$. We show that we can always empty the siphon $R_2 = \{p_3, p_4\}$. From M, first fire t_4 a number $M(p_3)$ times, thus reaching a marking M_1 where $M_1(p_3) = 0$. If $M_1(p_4) = 0$, we are done. Otherwise, $M_1(p_4) > 0$. As $M_1(R_1) > 0$ and $M_1(p_3) = 0$, either $M_1(p_1) > 0$, or $M_1(p_2) > 0$. We can reach a marking M_2 where $M_2(p_1) > 0$, $M_2(p_4) > 0$ and $M_2(p_3) = 0$ either by doing nothing (in the case where $M_1(p_1) > 0$), or by firing t_1 (in the case where $M_1(p_2) > 0$). At M_2, the sequence $t_3 t_2 t_1$ is enabled. This sequence reduces the number of tokens in p_4 by one and leaves the number of tokens in p_1, p_2 and p_3 unchanged. Therefore we can repeat this sequence until we reach a marking M' where $M'(p_3) = M'(p_4) = 0$.

We have shown that from any marking M, we can reach a marking M' where R_1 or R_2 is empty. Since both R_1 and R_2 are proper siphons, they stay empty, thus some transition of $R_1^\bullet$ or $R_2^\bullet$ is dead at M'. Therefore there is no initial marking M such that (N, M) is live.

Problem 3 (10 credits)

a) Let N be a connected T-net. Prove that if N is strongly connected, then it has a positive S-invariant. (5 credits)

Assume that N is strongly connected. Then any place belongs to some circuit. By the structure of T-nets, any circuit γ of N defines an S-invariant I_γ of N where $I_\gamma(p) = 1$ if $p \in \gamma$ and $I_\gamma(p) = 0$ otherwise. By taking the sum of all invariants I_γ for all circuits γ of N, one obtains a positive S-invariant of N.

b) Let N be a connected T-net. Prove that if N has a positive S-invariant, then it is strongly connected. (5 credits)

Assume that N is connected and has a positive S-invariant. Therefore (N, M_0) is bounded for any marking M_0. Take a marking M_0 such that $M_0(\gamma) > 0$ for every circuit γ of N. The system (N, M_0) is live (Liveness Theorem for T-systems) and also bounded, therefore N is strongly connected (Result of Exercise 7.3 or Corollary 5.2.5).