Comparing place invariants and the marking equation

**Definition**

Let $N$ be a net and let $L, M$ be two markings of $N$. We say that $L$ and $M$ agree on all invariants if $I \cdot L = I \cdot M$ for every invariant $I$.

**Theorem**

Let $N$ be a net and let $L, M$ be two markings of $N$. $L$ and $M$ agree on all invariants if the marking equation $L = M + X$ has a solution.
When \( X \in \mathbb{Q} \),

**Proof:**

(\( \Leftarrow \)) Easy

(\( \Rightarrow \)) Let \( V_c \) be the vector space generated by the columns of \( C \).

Let \( V_p \) be the vector space of \( p \)-invariant

By definition of \( p \)-invariant we have

\[ X \in V_p \iff X \cdot y = 0 \text{ for every } y \in V_c \]

A well-known theorem of linear algebra yields:

\[ X \in V_c \iff X \cdot y = 0 \text{ for every } y \in V_p \]
Since \( Y \cdot L = Y \cdot M \) holds for every \( Y \in V_p \), we have
\[
Y \cdot (L - M) = 0 \quad \text{for every} \quad Y \in V_p,
\]
and so, by the theorem above,
\[
(L - M) \in V_c.
\]
So \( (L - M) \) is a linear combination of the columns of \( C \), which means
\[
(L - M) = C \cdot x_0 \quad \text{for some} \quad x_0 \in \mathbb{Q}^{|T|},
\]
and so
\[
M + C \cdot x_0 = L.
\]