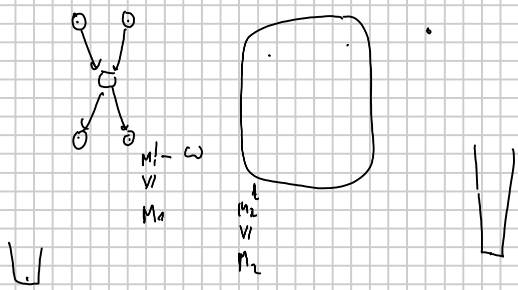


Abstract backwards overability algorithm

Definition Well-quasi-order (wqo)

Let A be a set. Let $\precsim \subseteq A \times A$ be a partial order on A.
The order $\precsim$ is a <u>well-quasi-order</u> if every infinite sequence $a_1 a_2 a_3 \ldots \in A^\omega$ contains an infinite chain $a_{i_1} \precsim a_{i_2} \precsim a_{i_3}, \ldots$

Examples of wqo's

- $\leq$ (pointwise) on $\mathbb{N}^n$
- Subword order on Σ^* for any finite alphabet Σ

$\precsim$ $\qquad w_1 \precsim w_2$

$$\begin{array}{c} aba \\ \curlyvee \\ aaabbabab \end{array}$$

Higman's Lemma

Definition

Let A be a set and let $\precsim$ be a wqo on A.
A set $X \subseteq A$ is <u>upward-closed wrt $\precsim$</u> if
$x \in X$ and $x \precsim y$ implies $y \in X$
A relation $\rightarrow \subseteq A \times A$ is <u>monotonic</u> if for all $x, y, x' \in A$

$$\begin{array}{ccc} x \rightarrow y & & x \rightarrow y \\ \curlyvee & \Rightarrow \exists y' & \curlyvee \quad \curlyvee \\ x' & & x' \rightarrow y' \end{array}$$

$$\begin{array}{l} (x, y) \in \rightarrow \\ \curlyvee \ \curlyvee \\ (x', y') \in \rightarrow \end{array}$$

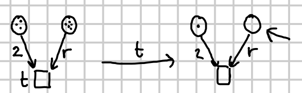
- $pre_{\rightarrow}(X) = \{ y \in A \mid y \rightarrow x \text{ and } x \in X \}$

Theorem Let A be a set, let $\precsim$ be a wqo on A, let $X_0 \subseteq A$ be upward closed wrt. $\precsim$, let $\rightarrow \subseteq A \times A$ be monotonic, then

$$pre^*_{\rightarrow}(X_0) = \bigcup_{i=1}^{j} pre^i_{\rightarrow}(X_0) \quad \text{for some } j$$

Applications:

- reset nets := nets + reset arcs

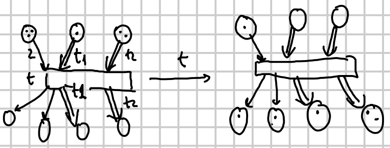


$$\begin{array}{ccc} M & \xrightarrow{t} & M' \\ \curlyvee & & \curlyvee \\ M'' & \xrightarrow{t} & M''' \end{array}$$

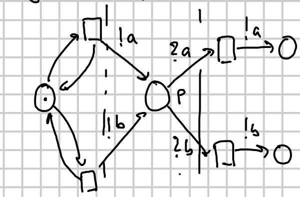
(transition is also enabled if there are no tokens in the source of a reset arc)

$\rightarrow$ is still monotonic

- transfer nets



- <u>lossy channel systems</u>



$(1, abb, \varepsilon, \varepsilon)$

$\ll$

$(2, bbabab, a, b)$

$\downarrow$

$(2, babb, a, \varepsilon)$

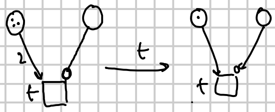
$M_0 \xrightarrow{!a} \xrightarrow{!b} \xrightarrow{!b} M$

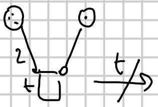
$M(p) = abb$

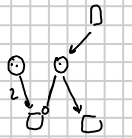
Turing-powerful extensions:

- channel systems (non-lossy)
- Petri nets with <u>inhibitor arcs</u>

inhibited for 2 tokens
not inhibited for
0, 1, ≥3 tokens!







Petri nets with (at least 2) inhibitor arcs are Turing-powerful