Automata and Formal Languages

Winter 2020/21

Syllabus

Course schedule

Lectures

Javier Esparza (esparza@in.tum.de)

Monday:	10:15 – 11:45
	Room: Interims I, Hörsaal 2, 5620.01.102
Thursday:	14:15: - 15:45
	Room: Interims I, Hörsaal 2, 5620.01.102

Exercises

Marijana Lazić (lazic@in.tum.de)
Chana Weil-Kennedy (chana.weilkennedy@in.tum.de)

Tuesday:	10:15 - 11:45
	Room: Interims I, Hörsaal 2, 5620.01.102

Course content

Automata on finite words

1. Automata classes and conversions
2. Minimization and reduction
3. Boolean operations and tests
4. Operations on relations
5. Operations on finite universes: decision diagrams
6. Automata and logic
7. Pattern-matching, verification, Presburger arithmetic

Course content

Automata on infinite words

8. Automata classes and conversions
9. Boolean operations
10. Emptiness check
11. Verification using temporal logic

Grading

Written exam

- end of the term
- planned as on-site exam
- 40 points
- retake at the beginning of next term (modulo Covid-19)

Exercises not graded!

Points	Grade
[36, 40]	1,0
[34, 36)	1,3
[32, 34)	1,7
[30, 32)	2,0
[28, 30)	2,3
[26, 28)	2,7
[24, 26)	3,0
[22, 24)	3,3
[19, 22)	3,7
[17, 19)	4,0
[11, 17)	4,3
[5, 11)	4,7
[0, 5)	5,0

Material

- Lecture notes available online
  Over 100 exercises with solutions
  No need to buy a book
- Many previous exams available online
- Slides available online
- Automata Tutor to help refresh

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www7.in.tum.de > Teaching
               > Automata
               > more info
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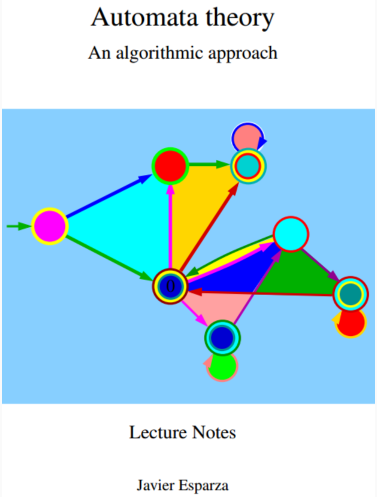

Automata theory: brief recap

Formal languages

An *alphabet* is a nonempty finite set of *letters*

e.g. $\{0, 1\}$, $\{a, b, \ldots, z\}$, $\{\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\}$, $\{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$

A *(finite) word* is a finite sequence of letters

e.g. 1001, $\mathtt{hello}$, $\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$, $\clubsuit\clubsuit\diamondsuit$, ε

A *language* is a set of words

e.g. $\{1, 10, 100, 1000, \ldots\}$, $\{aa, aba, abbba, \ldots\}$

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A *(finite) word* is a finite sequence of letters

e.g. $\mathtt{1001}$, $\mathtt{hello}$, $[\begin{smallmatrix}1\\0\end{smallmatrix}][\begin{smallmatrix}0\\1\end{smallmatrix}][\begin{smallmatrix}1\\0\end{smallmatrix}]$, $\clubsuit\clubsuit\diamondsuit$, ε

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e.g. $\{\mathtt{1}, \mathtt{10}, \mathtt{100}, \mathtt{1000}, \ldots\}$, $\{\mathtt{aa}, \mathtt{aba}, \mathtt{abbba}, \ldots\}$

Formal languages

Let $u = a_1 \cdots a_n$ and $v = b_1 \cdots b_m$ be words

Concatenation:

$$u \cdot v = uv = a_1 \cdots a_n b_1 \cdots b_m$$
$$\varepsilon \cdot u = u = u \cdot \varepsilon$$

Exponentiation: $u^0 = \varepsilon$, $u^{k+1} = u^k \cdot u$

e.g. $\mathtt{a}^0 = \varepsilon$, $\mathtt{a}^1 = \mathtt{a}$, $(\mathtt{hallo})^2 = \mathtt{hallohallo}$,
$\mathtt{1}^5 = \mathtt{11111}$, $\varepsilon^{1000} = \varepsilon$, $\mathtt{ab} \cdot \mathtt{cde} = \mathtt{abcde}$

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Formal languages

Let L and L' be languages over alphabet Σ

Concatenation: $L \cdot L' = LL' = \{u \cdot v : u \in L, v \in L'\}$

Exponentiation: $L^0 = \{\varepsilon\}$, $L^{k+1} = L^k \cdot L$

Iteration: $L^* = L^0 \cup L^1 \cup L^2 \cup \cdots$

Complement: $\overline{L} = \Sigma^* \setminus L$

e.g. $$\begin{aligned} \{\mathrm{aa}, \mathrm{bb}\} \cdot \{\mathrm{c}, \mathrm{d}\} &= \{\mathrm{aac}, \mathrm{aad}, \mathrm{bbc}, \mathrm{bbd}\}, \\ \{\mathrm{aa}, \mathrm{b}\}^* &= \{\varepsilon, \mathrm{aa}, \mathrm{b}, \mathrm{aaaa}, \mathrm{aab}, \ldots\} \end{aligned}$$

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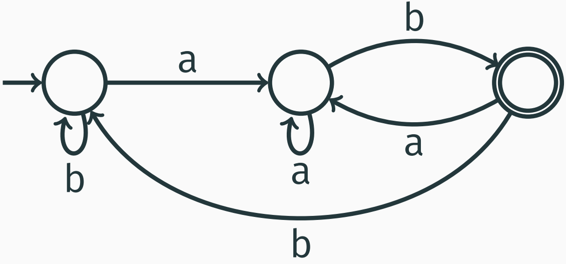
Deterministic finite automata (DFA)

- *States*: nonempty finite set Q
- *Alphabet*: nonempty finite set Σ
- *Transitions*: $\delta : Q \times \Sigma \to Q$
- *Initial state*: $q_0 \in Q$
- *Final states*: $F \subseteq Q$



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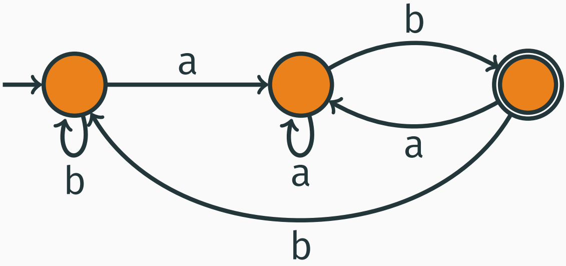

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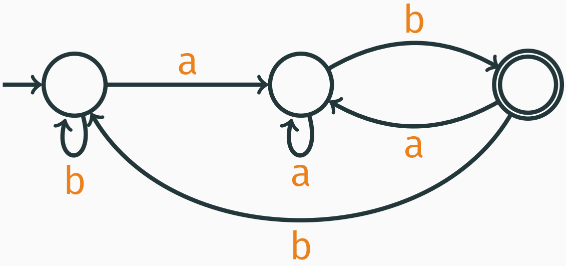

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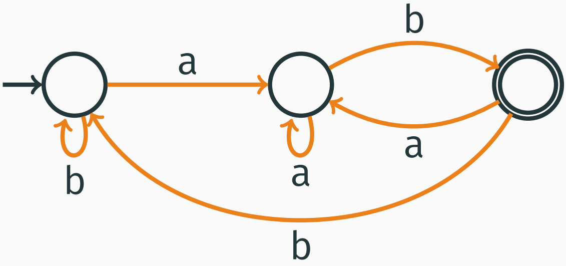

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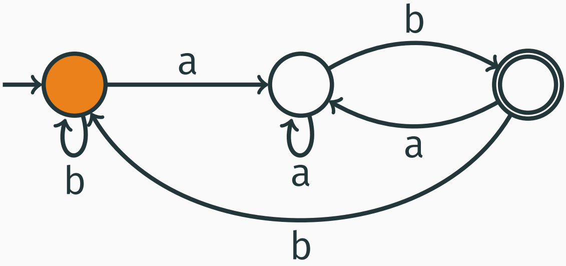

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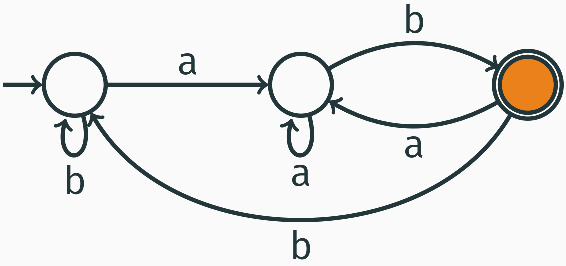
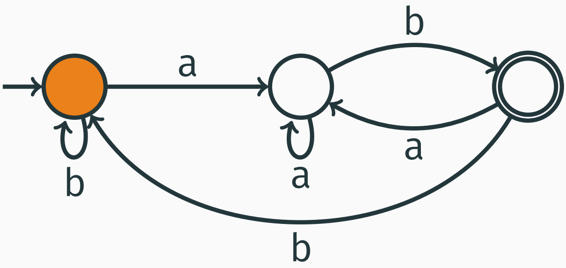


w = aabab



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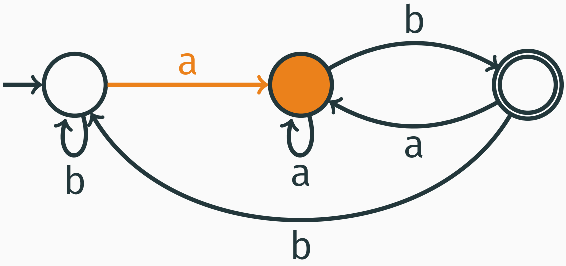

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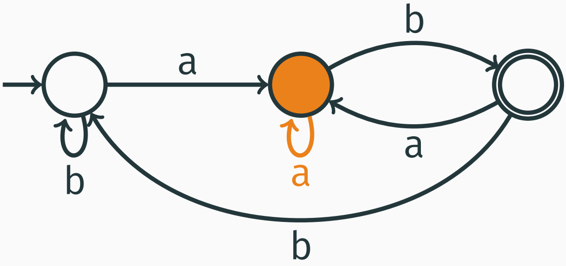

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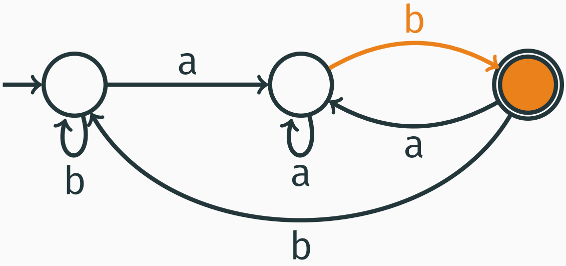

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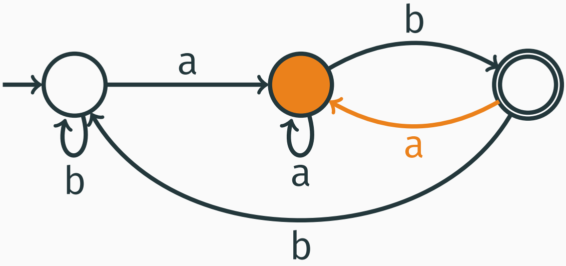

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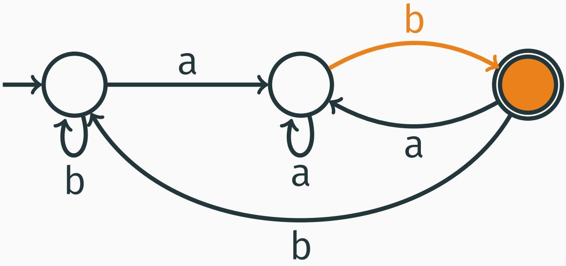
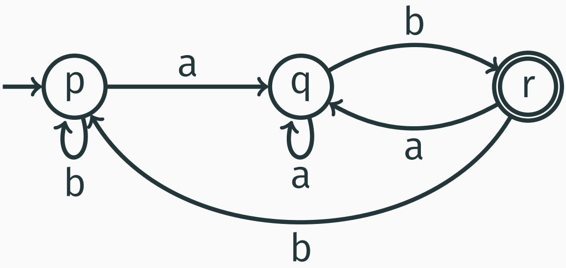
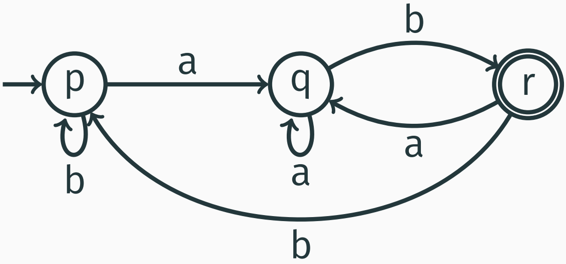


$$p \xrightarrow{a} q \xrightarrow{a} q \xrightarrow{b} r \xrightarrow{a} q \xrightarrow{b} r$$



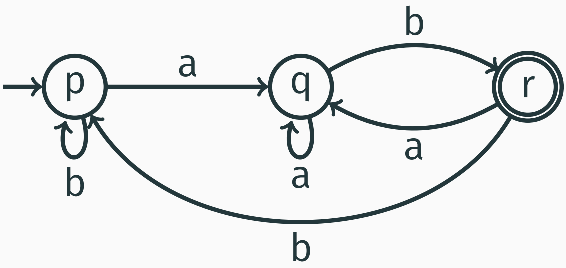
Deterministic finite automata (DFA)

$$p \xrightarrow{\quad aabab \quad} r$$



Deterministic finite automata (DFA)

$$L(A) = \{w \in \Sigma^* : \exists q \in F \text{ s.t. } q_0 \xrightarrow{w} q\}$$



Deterministic finite automata (DFA)

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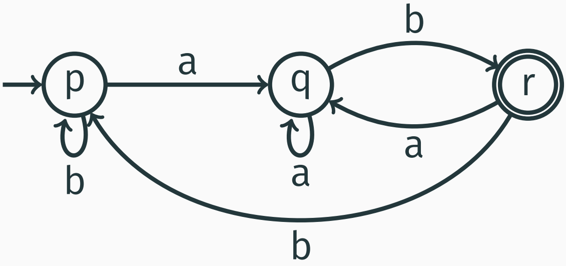
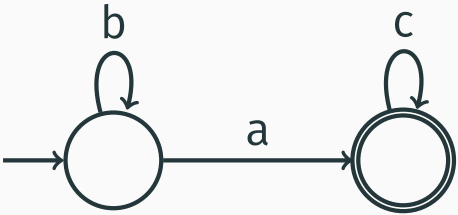


Transition function δ defined *on every* input



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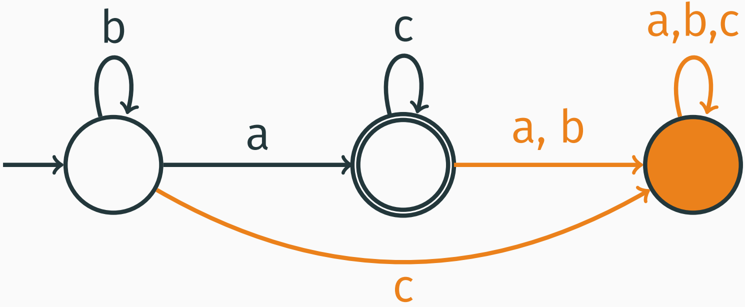
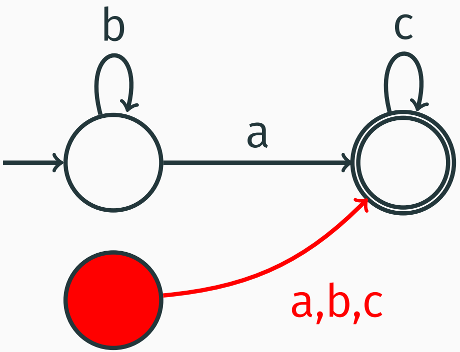


Every state *reachable* from initial state



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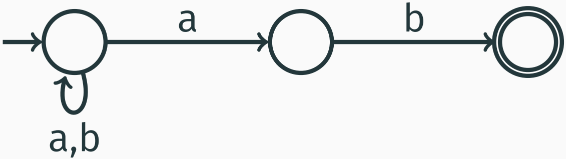

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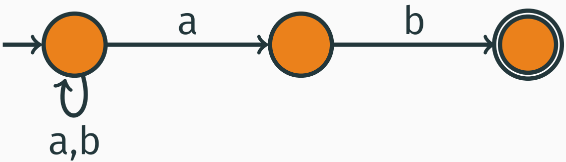

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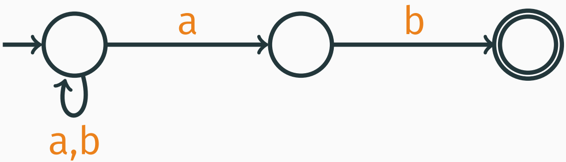

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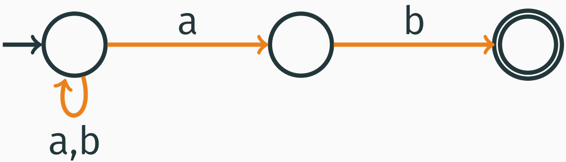

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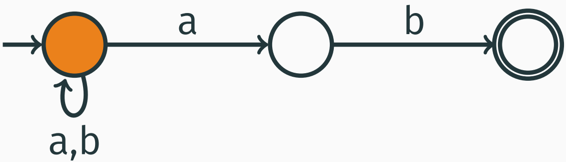

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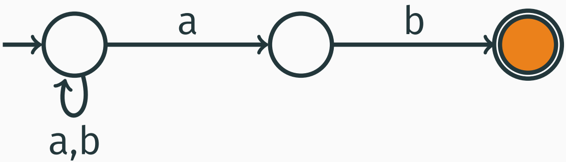

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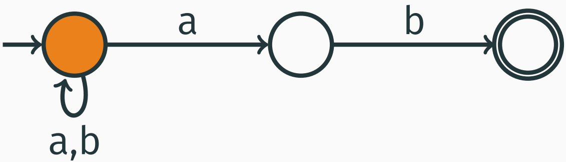

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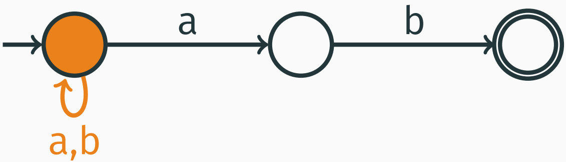

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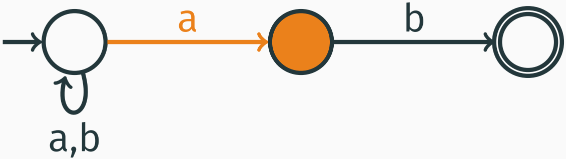

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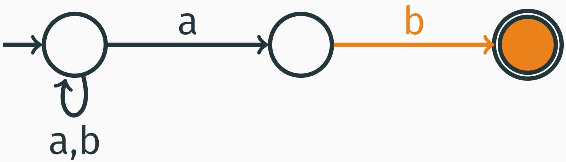

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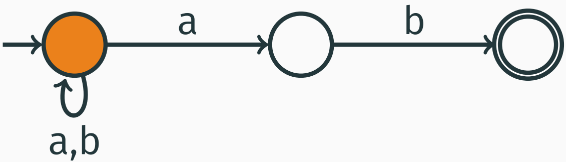
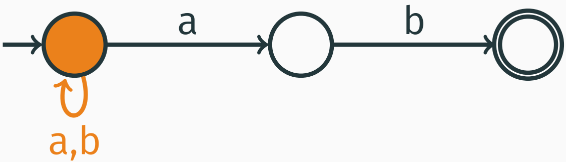


Nondeterministic finite automata (NFA)

w = aab



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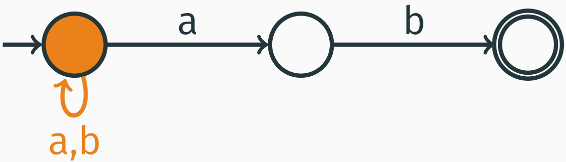

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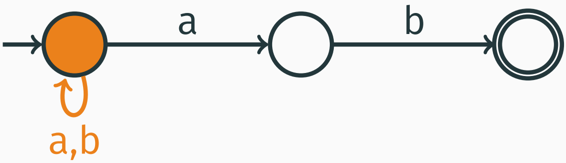
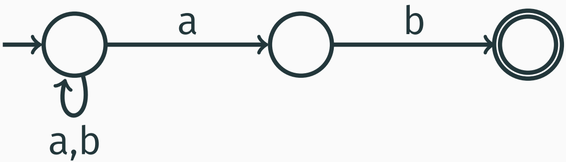


Nondeterministic finite automata (NFA)

$$p_0 \xrightarrow{a_1} p_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} p_n$$

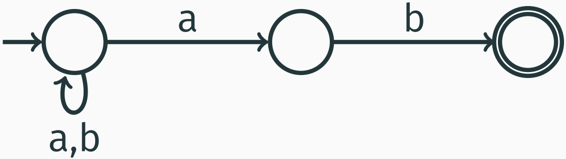
$$\Updownarrow$$

$$p_i \in \delta(p_{i-1}, a_i) \text{ for every } 0 < i \leq n$$



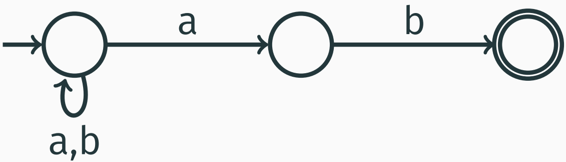
Nondeterministic finite automata (NFA)

$$L(A) = \{w \in \Sigma^* : \exists q_0 \in Q_0, q \in F \text{ s.t. } q_0 \xrightarrow{w} q\}$$



Nondeterministic finite automata (NFA)

$$L(A) = \{w \in \Sigma^* : \quad w \text{ ends with ab} \quad \}$$



Regular expressions

$$r \ ::= \ \emptyset \ \mid \ \varepsilon \ \mid \ a \ \mid \ r_1 r_2 \ \mid \ r_1 + r_2 \ \mid \ r^*$$

Regular expressions

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$$\begin{aligned} L(\emptyset) &= \emptyset & \quad L(r_1\, r_2) &= L(r_1) \cdot L(r_2) \\ L(\varepsilon) &= \{\varepsilon\} & \quad L(r_1 + r_2) &= L(r_1) \cup L(r_2) \\ L(a) &= \{a\} & \quad L(r^*) &= L(r)^* \end{aligned}$$

Regular expressions

$$L(\,(a+b)^*ab\,) = \{w \in \{a,b\}^* : w \text{ ends with } \texttt{ab}\}$$

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More examples

$$L = \{w \in \{\mathrm{a}, \mathrm{b}\}^* : w \text{ contains } \mathrm{aaa}\}$$

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Regular expression?

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Regular expression?

$$(\mathrm{a} + \mathrm{b})^*\mathrm{aaa}(\mathrm{a} + \mathrm{b})^*$$

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NFA?

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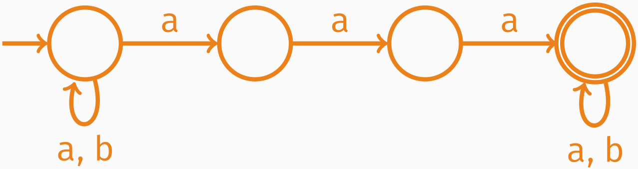

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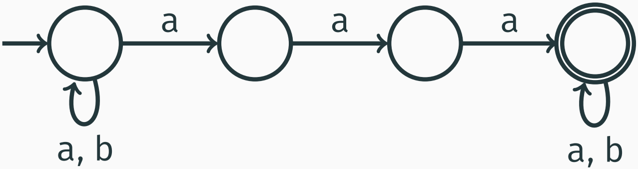
DFA?



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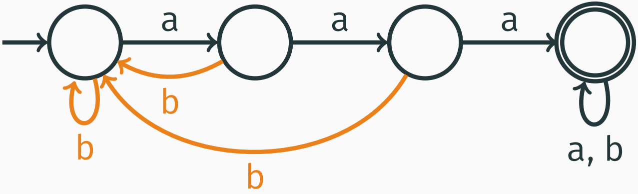
DFA?



More examples

$$L = \{w \in \{0,1\}^* : w \text{ contains } \begin{array}{l} \text{an even number of 0 or} \\ \text{an odd number of 1} \end{array} \}$$

More examples

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Regular expression?

More examples

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Regular expression?

$$(1^*01^*0)^*1^* + (0^*10^*1)^*0^*10^*$$

More examples

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NFA?

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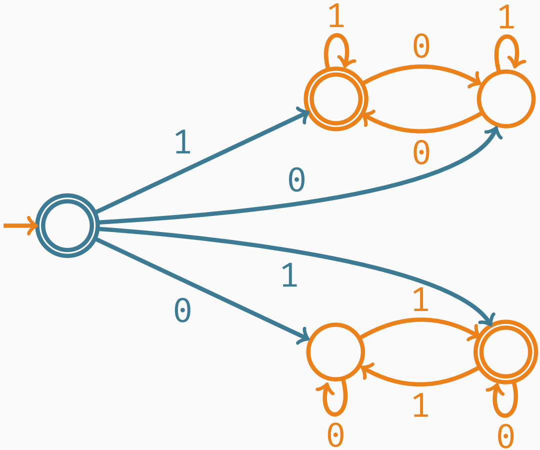
NFA?



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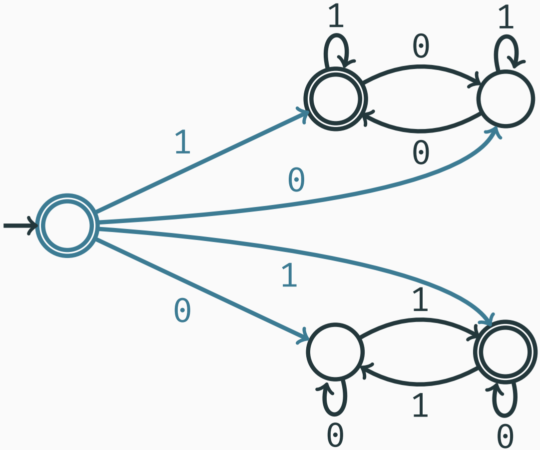

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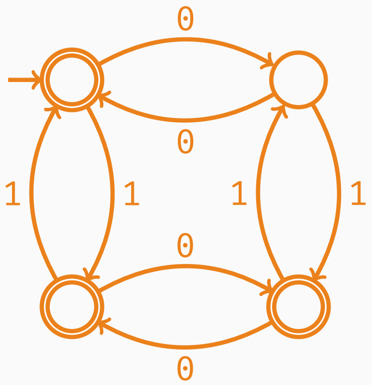
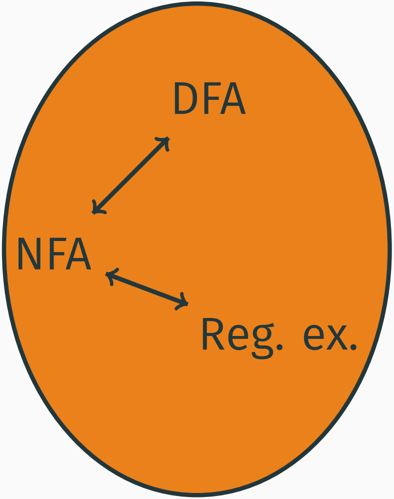
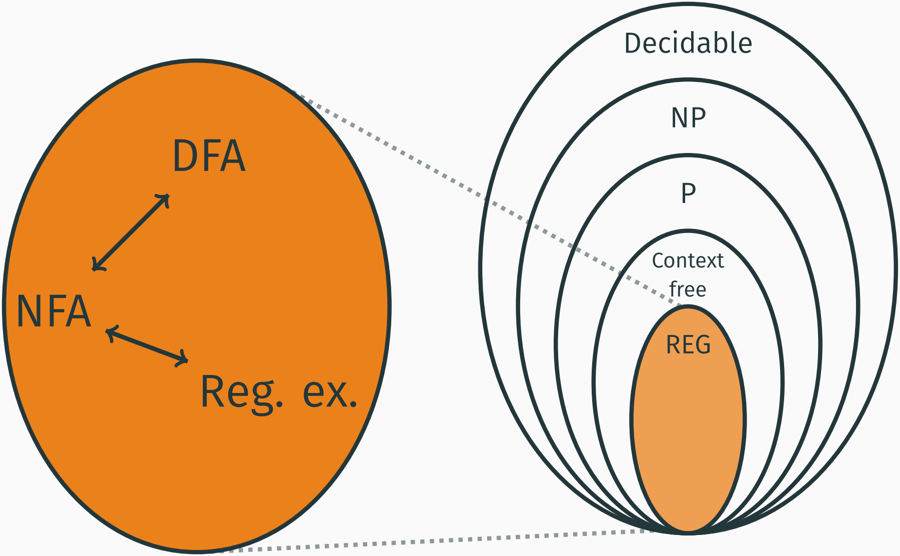
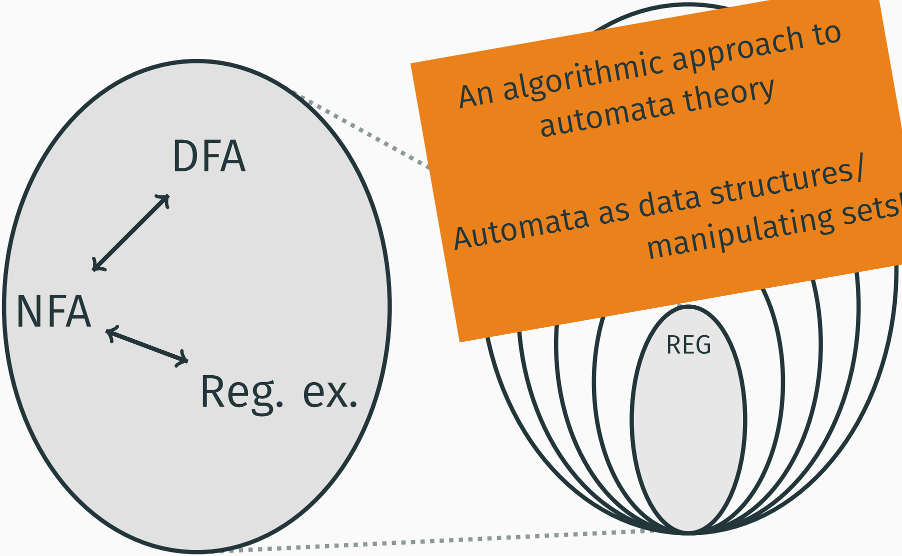


Regular languages



Regular languages





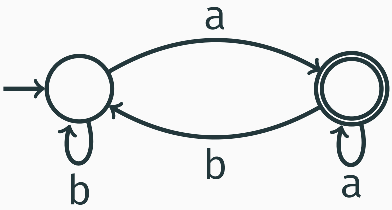
An algorithmic approach to automata theory

Automata as data structures/ manipulating sets!

Beyond finite words

An *infinite word* is an infinite sequence $a_0 a_1 a_2 \cdots$ over some Σ

A *Büchi automaton* is ”as an NFA”, but accepts infinite words



Büchi automata

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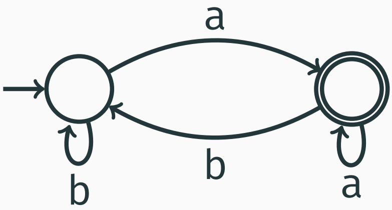

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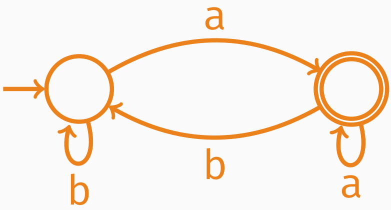

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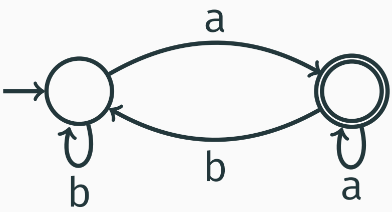
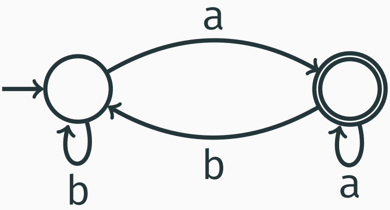


$L_\omega(A) = \{w \in \{\mathrm{a}, \mathrm{b}\}^\omega : w \text{ contains infinitely many a}\}$

An *i*… …over some Σ

Coming later this semester!

A *Bü*… …te words



$$L_\omega(A) = \{w \in \{\mathsf{a}, \mathsf{b}\}^\omega : w \text{ contains infinitely many } \mathsf{a}\}$$